Metal Forming Processes (ME5807)

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Modelling Techniques for Metal Forming

- Slab Method
- Upper and Lower Bound Techniques
- Slip Line Field Theory
- Finite Element Method

Slab Method

The distribution of contact pressure between the tool and work, and the working stress is obtained through simplification of the stress state. The entire deformation region is divided into a finite number of elements (slabs) and an equilibrium equation for a typical element is solved subjected to the given stress boundary conditions. The analysis is based on the following assumptions:

- The stresses on a element surface, perpendicular to the flow direction are in princial directions and are not permitted to vary on the surface
- The external frictional stresses do not affect the internal stress distribution.

Open Die Forging

Open Die forging is carried out between flat dies or dies of very simple shape. Often open die forging is used to preform the workpiece for the closed die forging.

Consider the case of open die forging or upsetting operation in which a rectangular block is compressed between to parallel plates. The maximum force/load required to perform the upsetting operation can be obtained using the slab method.

- Low friction
- High friction
- Mixed friction

The following assumptions are taken valid for all three cases:

- The deformation occurs under plan strain condition. (The thickness of the strip is much smaller than the other dimensions.)
- The coefficient of friction (μ) remains constant throughout the interface.
- Stresses are uniformly distributed across the entire cross-section.
- The maximum forging force occurs at the end point of forging operation.
- The entire workpiece is in plastic state during the process.

Low (Slipping) Friction Condition

Equilibrium for RHS element in flow direction,

$$(\sigma_x + d\sigma_x)h - \sigma_x h - 2\mu p dx = 0 \quad (1)$$

Where,
$\sigma_x =$ stress acting in the x-direction
$d\sigma_x =$ incremental stress acting in the x-direction
$p =$ normal stress / pressure acting on the slab-work inreface.

For the LHS element,

$$(\sigma_x + d\sigma_x)h - \sigma_x h + 2\mu p dx = 0 \quad (2)$$

After neglecting the higher order terms, the above two equations can be written as

$$h d\sigma_x \mp 2\mu p dx = 0 \quad (3)$$

In this equation, Minus sign $(-)$ correspond to the RHS element and Plus sign $(+)$ correspond to the LHS element

Low (Slipping) Friction Condition

The Von Mises criterion in plain strain condition is Equilibrium for RHS element in flow direction,

$$\sigma_1 - \sigma_3 = 2k \tag{4}$$

In the present case, $\sigma_1 = \sigma_x$ and $\sigma_3 = -p$. Therefore, yield criterion can be expressed as,

$$\sigma_x + p = 2k; \qquad d\sigma_x = -dp \tag{5}$$

Substituting in equation 3 gives,

$$-hdp \mp 2\mu p dx = 0 \tag{6}$$

Low (Slipping) Friction Condition

$$-hdp \mp 2\mu p dx = 0$$

Rearranging both sides

$$\frac{dp}{p} = \mp \frac{2\mu}{h} dx \tag{7}$$

Integrating both sides,

$$\ln p = \mp \frac{2\mu}{h} x + Constant \tag{8}$$

It can be written as,

$$p = Ce^{\mp \frac{2\mu}{h} x} \tag{9}$$

At $x = \pm\frac{b}{2}$, $\sigma_x = 0$. Therefore at $x = \frac{b}{2}$

$$(p)_{b/2} = Ce^{\mp\frac{2\mu}{h}x} \tag{10}$$

from the yield criterion,

$$(p)_{b/2} = 2k - (\sigma_x)_{b/2} = 2k \tag{11}$$

The pressure acting on the tool-work interface (on RHS) can be obtained as

$$\frac{p}{2k} = e^{\frac{2\mu}{h}(\frac{b}{2}-x)} \tag{12}$$

The pressure acting on the tool-work interface (on LHS) can be obtained as

$$\frac{p}{2k} = e^{\frac{2\mu}{h}(\frac{b}{2}+x)} \tag{13}$$

Low (Slipping) Friction Condition

Equation (12) and (13) indicate that the pressure incresease exponentially as one moves inward from $p = 2k$ at both ends. The maximum pressure value is at center ($x = 0$) is,

$$(\frac{p}{2k})_{max} = e^{\frac{\mu b}{h}} \tag{14}$$

For very low values of μ, the above expression can be written as,

$$(\frac{p}{2k})_{max} = 1 + \frac{\mu b}{h} \tag{15}$$

Low (Slipping) Friction Condition

The pressure ditribution for various values of μ indicates friction hill in all cases.

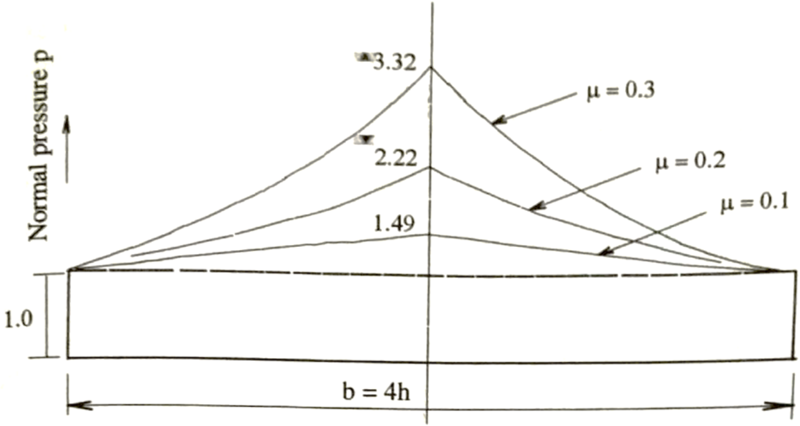

Figure: Distribution of Normal Pressure under low friction conditions

High (Sticking) Friction Condition

The equilibrium equation can be written as.

$$\sigma_x \mp 2kdh = 0 \tag{16}$$

The yield criterion can be expressed as

$$\sigma_x + p = 2k$$

Differentiation of yield condition results in,

$$d\sigma_x = -p$$

By substituting the above equation in the equilibrium equation reduces it to

$$\frac{dp}{2k} = \mp \frac{dx}{h} \tag{17}$$

High (Sticking) Friction Condition

The integration of equation (17),

$$\frac{p}{2k} = \mp\frac{x}{h} + Constant \tag{18}$$

Constant C can be obtained by the boundary condition,

$$\sigma_x = 0; \quad x = \pm\frac{b}{2}$$

From the yield condition, pressure value p = 2k at the edges. Pressure expression can obtained as

$$\frac{p}{2k} = 1 + \frac{\frac{b}{2} \mp x}{h} \tag{19}$$

and the maximum value of pressure at $x = 0$ is obtained as;

$$(\frac{p}{2k})_{max} = 1 + \frac{b}{2h} \tag{20}$$

High (Sticking) Friction Condition

The pressure ditribution for various values of μ under sticking friction condition.

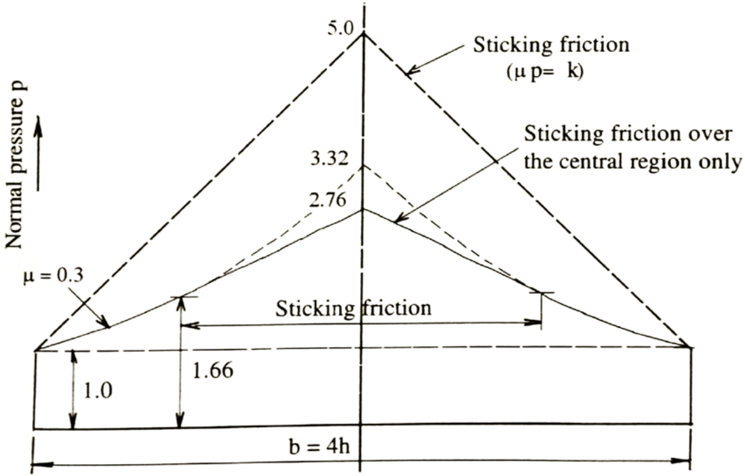

Figure: Distribution of Normal Pressure under sticking friction

The equilibrium equation for mixed friction case can be written as,

$$hd\sigma_x \mp 2\tau dx = 0 \tag{21}$$

Where, $\tau = \mu p$ if $\mu p < k$ and $\tau = k$ if $\mu p \geq k$.
von Mises yield criterion gives

$$\sigma_x + p = 2k$$

and

$$d\sigma_x = -dp$$

Substituting the above equation in the equilibrium equation gives

$$hdp \mp 2\tau dx = 0 \tag{22}$$

Mixed Friction Condition

Assuming that the transition from sticking to slipping occurs at point x_1, the transition point can be obtained from the condition,

$$\mu p = k; \quad (\frac{p}{2k})_{x_1} = \frac{1}{2\mu} \tag{23}$$

Using equation (12) for slipping friction condition, the normal pressure at x_1 is,

$$(\frac{p}{2k})_{x_1} = e^{[\frac{2\mu}{h}(\frac{b}{2} - x_1)]} \tag{24}$$

The normal pressure at $x = x_1$ will be same for both slipping and sticking friction conditions, Therefore,

$$e^{[\frac{2\mu}{h}(\frac{b}{2} - x_1)]} = \frac{1}{2\mu} \tag{25}$$

Taking logarithm on both sides, one can get,

$$\frac{2\mu}{h}\left(\frac{b}{2} - x_1\right) = ln\left(\frac{1}{2\mu}\right)$$

or

$$x_1 = \frac{b}{2} - \frac{h}{2\mu} ln\left(\frac{1}{2\mu}\right) \qquad (26)$$

From the point x_1 to outer edge the normal pressure value will be governed by the equation for slipping friction and from center to x_1 it will be governed by the equation for sticking friction. Normal Pressure for sticking friction region on RHS (equation 18) is,

$$\frac{p}{2k} = -\frac{x}{h} + Constant$$

The boundary condition at $x = x_1$ gives,

$$\frac{p}{2k} = \frac{1}{2\mu}$$

Using the above two equations,

$$Constant = \frac{1}{2\mu} + \frac{x_1}{h} \tag{27}$$

Thus for values of normla pressure between 0 and x_1, at any value of x on RHS can be obtained as,

$$(\frac{p}{2k}) = \frac{1}{2\mu} + \frac{x_1 - x}{h} \tag{28}$$

A similar procedure can be used to derive tge expression for LHS.

Load calculation

The total forging force per unit length of the workpiece is obtained by integrating the pressure on the interface area. The mean forging pressure can be obtained as;

$$p_{avg} = \int_0^{\frac{b}{2}} \frac{p}{b/2} dx \tag{29}$$

The total forging load P per unit length can be obtained as;

$$P = bp_{avg} \tag{30}$$

Strip Drawing

Strip Drawing – Slab Method

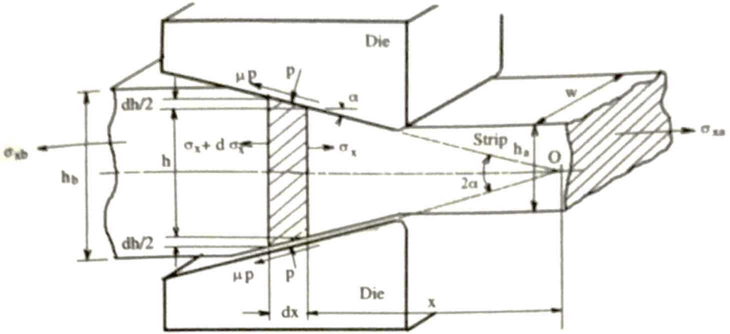

Figure: Stresses on an element during strip drawing with wedge-shaped dies

Equilibrium of forces acting along the longitudinal direction gives,

$$(\sigma_x + d\sigma_x)(h + dh)w - \sigma_x hw + 2p(w\frac{dx}{cos\alpha})sin\alpha + 2\mu p(w\frac{dx}{cos\alpha})cos\alpha = 0 \quad (31)$$

Neglecting the product $d\sigma_x.dh$ and eliminating w gives,

$$\sigma_x\, dh + h\, d\sigma_x + 2p\, dx\, tan\alpha + 2\mu p\, dx = 0 \quad (32)$$

Since,

$$h = 2x\, tan\alpha$$

and

$$dh = 2dx\, tan\alpha$$

Equation (32) reduces to

$$h\, d\sigma_x + [\, \sigma_x + p(\, 1 + \mu \cot \alpha)\,]\ \ dh = 0$$

or

$$h\, d\sigma_x + [\, \sigma_x + p(\, 1 + B)\,]\ \ dh = 0 \qquad (33)$$

Where,

$$B = \mu \cot \alpha$$

Equilibrium of forces in the vertical direction ,i.e., in the direction perpendicular to the direction of drawing gives,

$$\sigma_y \, dx + p\left(\frac{dx}{\cos\alpha}\right)\cos\alpha - \mu p\left(\frac{dx}{\cos\alpha}\right)\sin\alpha = 0 \tag{34}$$

or

$$\sigma_y \, dx + p \, dx - \mu p \tan\alpha \, dx = 0$$

after simplification

$$\sigma_y \, dx = -p(1 - \mu \tan\alpha)\, dx \tag{35}$$

For small values of μ and α, $\mu \tan\alpha$ can be ignored. this gives,

$$\sigma_y \approx -p \tag{36}$$

and $\sigma_y(\sigma_3)$ become a principal stress.

For plane strain deformation, von Mises yield criterion reduces to ($\sigma_1 - \sigma_3 = Y$) . Thus,

$$\sigma_x + p = Y \tag{37}$$

Substituting this in equation (33) and rearranging gives,

$$\frac{d\sigma_x}{B\sigma_x - Y(1 + B)} = \frac{dh}{h} \tag{38}$$

Taking μ and Y to be constants, integration of this equation gives

$$\frac{1}{B} \ln[B\sigma_x - Y(1 + B)] = \ln h + Constant \;\; C'$$

or

$$B\sigma_x - Y(1 + B) = C\,h^B \tag{39}$$

Where $C = e^{C'B}$.

If no back tension is applied, boundary condition at the die entry gives,

$$\sigma_x = \sigma_{xb} = 0; \quad h = h_b$$

Thus,

$$-Y(1 + B) = C\, {h_b}^B$$

and

$$\sigma_x = \frac{1}{B}[-Y(1 + B)\left(\frac{h}{h_b}\right)^B + Y(1 + B)] \qquad (40)$$

or

$$\frac{\sigma_x}{Y} = \frac{1 + B}{B}[1 - \left(\frac{h}{h_b}\right)^B] \qquad (41)$$

The drawing stresses σ_{xa} is the value of σ_x at the die exit, where $h = h_a$. Therefore, the drawing stress required for strip drawing.

$$\frac{\sigma_{xa}}{Y} = \frac{1+B}{B}[1-(\frac{h_a}{h_b})^B] \tag{42}$$

Using equation (36), the die pressure at any point along the interface can be evaluated as,

$$(\frac{p}{Y})_x = 1 - \frac{1+B}{B}[1-(\frac{h}{h_b})^B] \tag{43}$$

Wire/Rod Drawing

Wire/Rod Drawing

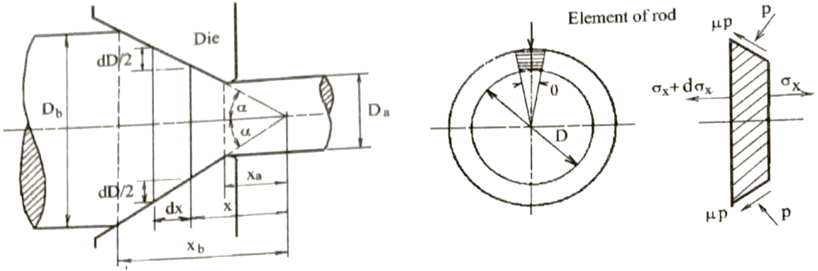

Figure: Stresses acting on an element during wire/rod drawing through a conical die

The stress equilibrium equation for the element in the axial direction can be written as

$$(\sigma_x + d\sigma_x)\frac{\pi}{4}(D+dD)^2 - \sigma_x \frac{\pi}{4}D^2 + p(\pi D \frac{dx}{\cos\alpha})\sin\alpha + \mu p(\pi D \frac{dx}{\cos\alpha})\cos\alpha = 0 \quad (44)$$

Here σ_x is the axial stress, p is the die pressure and μ is the co-efficient of friction.

On Simplification this gives,

$$\frac{\sigma_x D\, dD}{2} + \frac{D^2 d\sigma_x}{4} + pD\, dx \tan\alpha + \mu p D\, dx = 0 \quad (45)$$

Using $dD = 2dx \tan\alpha$ gives

$$2\sigma_x\, dD + D\, d\sigma_x + 2p\, dD + 2\mu p\, dD \cot\alpha = 0$$

or

$$D\, d\sigma_x + 2[\sigma_x + p(1 + \mu \cot\alpha)]\; dD = 0 \quad (46)$$

Similarly, radial equilibrium gives,

$$\sigma_r(\pi D\, dx) = -p(\pi D \frac{dx}{\cos\alpha}) \cos\alpha + \mu p(\pi D \frac{dx}{\cos\alpha}) \sin\alpha \quad (47)$$

On Simplification gives the radial stresse as,

$$\sigma_r = -p(1 - \mu \tan\alpha) \quad (48)$$

In wire drawing α is usuallary small, seldom exceeding 12^o. Therefore,

$$\sigma_r \approx -p \quad (49)$$

The principal stresses in the wire drawing are $\sigma_1 = \sigma_x$, and $\sigma_2 = \sigma_3 = \sigma_r = -p$ (cylindrical state of stress).

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$$

gives,

$$\sigma_x + p = Y \quad (50)$$

The axial equilibrium can now be written as,

$$\frac{d\sigma_x}{B\sigma_x - Y(1+B)} = 2\frac{dD}{D} \tag{51}$$

Where, $B = \mu\ cot\ \alpha$. Assuming B and Y to be constants, integration of this equation gives

$$\frac{1}{B}\ \ln[\,B\sigma_x - Y(1+B)\,] = 2\ \ln\ D + Constant$$

or

$$B\sigma_x - Y(1+B) = C\,D^{2B} \tag{52}$$

Where C is a constant.
Assuming no back tension, the axial stress at the entry is zero. Therefore,

$$\sigma_x = \sigma_{xb} = 0; \quad D = D_b$$

This condition gives,

$$C = -Y(1+B)\,/D_b^{2B}$$

The equation (51) now gives,

$$\frac{\sigma_x}{Y} = \frac{1+B}{B}\left[1 - \left(\frac{D}{D_b}\right)^{2B}\right] \tag{53}$$

The drawing stress σ_{xa} (the axial stress at the exit end) can now be evaluated as

$$\frac{\sigma_{xa}}{Y} = \frac{1+B}{B}\left[1 - \left(\frac{D_a}{D_b}\right)^{2B}\right] \tag{54}$$

In terms of reduction in area,

$$r = \frac{\pi}{4}\left(D_b^2 - D_a^2\right) / \frac{\pi}{4} D_b^2$$

or

$$r = 1 - \left(\frac{D_a^2}{D_b^2} \right)$$

The drawing stress can now be written as

$$\frac{\sigma_{xa}}{Y} = \frac{1+B}{B} \left[1 - (1-r)^{B} \right] \tag{55}$$

Tube Drawing

Taking, both the die pressure and plug pressure to be equal to p, the axial equilibrium equation can be written as

$$(\sigma_x + d\sigma_x)(h + dh)\pi D - \sigma_x h \pi D + \int_0^{2\pi} p\left(\frac{dx}{\cos\alpha}\frac{D}{2}d\theta\right)\sin\alpha + \int_0^{2\pi} p\left(\frac{dx}{\cos\beta}\frac{D}{2}d\theta\right)\sin\beta + \int_0^{2\pi} \mu_1 p\left(\frac{dx}{\cos\alpha}\frac{D}{2}d\theta\right)\cos\alpha + \int_0^{2\pi} \mu_2 p\left(\frac{dx}{\cos\beta}\frac{D}{2}d\theta\right)\cos\beta = 0 \qquad (56)$$

This on simplification gives

$$(\sigma_x\, dh + h\, d\sigma_x)\,\pi D + p\pi D(\tan\alpha - \tan\beta)\,dx + p\pi D(\mu_1 + \mu_2)\ dx = 0$$

or

$$(\sigma_x\, dh + h\ + p\, dh[1 - \frac{(\mu_1 + \mu_2)}{(\tan\alpha - \tan\beta)}]\ = 0 \qquad (57)$$

since the net change in wall thickness

$$dh = dx\tan\alpha - dx\tan\beta \qquad (58)$$

Now, equation (57) can be written as

$$h\, d\sigma_x + [\sigma_x + p(1 + B^*)]\ dh = 0 \qquad (59)$$

where plug drawing parameter,

$$B^* = \frac{\mu_1 + \mu_2}{\tan\alpha - \tan\beta}$$

As in the case of wire drawing, if the contribution of friction to die pressure is small, the principal stresses can be taken as,

$$\sigma_1 = \sigma_x; \quad \sigma_2 = -p$$

and von Mises yield criterion gives,

$$\sigma_x + p = Y$$

Substituting in equation (59) gives,

$$\frac{d\sigma_x}{B^*\sigma_x - Y(1 + B^*)} = \frac{dh}{h} \tag{60}$$

Assuming μ and Y to be constant, integration of the euqation (60) gives

$$\frac{1}{B^*} \ln B^* \sigma_x - Y(1 + B^*) = \ln h + Constant$$

or

$$B^* \sigma_h - Y(1 + B^*) = C h^{B^*}$$

Where C is constant.
Assuming no back tension , the condition,

$$\sigma_x = \sigma_{xb} = 0; \quad h = h_b$$

gives the contant of integration,

$$C = -Y(1 + B^*) h_b^{-B^*}$$

Equation (58) now becomes

$$\frac{\sigma_x}{Y} = \frac{1 + B^*}{B^*}[1 - (\frac{h}{h_b})^{B*}] \qquad (61)$$

The drawing stress can be evaluated as the axial stress at the die exit where $h = h_b$ as

$$\frac{\sigma_{xa}}{Y} = \frac{1 + B^*}{B^*}[1 - (\frac{h_a}{h_b})^{B*}] \qquad (62)$$

Extrusion

Extrusion of Round Bar

The differential equation for axial equlibrium is obtained as

$$D\, d\sigma_x + 2[\, \sigma_x + p(\, 1 + \mu\, cot\alpha)\,]\;\; dD = 0 \tag{63}$$

This equation is same as equation (45).
In wire drwaing is was assummed that $\sigma_r \approx -p$ since α is seldom greater than 12^o. This may not true is extrusion process where α may be as high as 60^o and taking $\sigma_r \approx -p$ may introduce substantial inaccuracy in the estimated load. However, wherever this assumption can be considered as valid, the yield criterion can be simplified as

$$\sigma_x + p = Y \tag{64}$$

as in wire drawing. The axial equilibrium equation can, therefore, be written as

$$\frac{d\sigma_x}{B\sigma_x - Y(\, 1 + B)} = 2\frac{dD}{D} \tag{65}$$

Taking B and Y to be constant

$$B\sigma_x - Y(1 + B) = CD^{2B} \tag{66}$$

The constant C now can be evaluated using the boundary condition, $\sigma_x = 0$ at exit end where $D = D_a$. This gives,

$$\frac{\sigma_x}{Y} = \frac{1 + B}{B}[1 - (\frac{D}{D_a})^{2B}] \tag{67}$$

and the extrusion pressure σ_{xb} acting at the entrance to the die as

$$\frac{\sigma_{xb}}{Y} = \frac{1 + B}{B}[1 - (\frac{D_b}{D_a})^{2B}] \tag{68}$$

Extrusion of Flat Strip

Extrusion of Flat Strip

The equilibrium of forces acting in the longitudinal direction in this case gives

$$h\, d\sigma_x + [\, \sigma_x + p(\, 1 + B)\,] \;\; dh = 0 \tag{69}$$

Using yield criterion, $\sigma_x + p = 0$, and integration with boundary conditions $\sigma_{xa} = 0$ at $h = h_a$ gives the extrusion pressure as

$$\frac{\sigma_{xb}}{Y} = \frac{1 + B^*}{B^*}[\, 1 - (\, \frac{h_a}{h_b}) \,^B] \tag{70}$$

$$\frac{\sigma_{xb}}{Y} = \frac{1 + B^*}{B^*}[\, 1 - r^B] \tag{71}$$

End of Module