Introduction to Engineering Optimization (ME6806)

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Module 4

Multi-variable Optimization Algorithms

Outlines

- **Optimality Criteria**
- **Direct Search Methods**
  - Nelder and Mead (Simplex Search)
  - Hook and Jeeves (Pattern Search)
  - Powell’s Method (The Conjugated Direction Search)
- **Gradient Based Methods**
  - Cauchy’s Method
  - Newton’s Method
  - Modified Newton’s Method
  - Marquardt’s Method
  - Conjugate Gradient Method
  - Quasi-Newton Method
  - Trust Regions
  - Gradient-Based Algorithm
  - Numerical Gradient Approximations

Hook and Jeeves (Pattern Search) Method

Exploratory Moves

Given a specified step size, which may be different for each coordinate direction and change during the search, the exploration proceeds from an initial point by the specified step size in each coordinate direction.

If the function value does not increase, the step is considered successful. Otherwise, the step is retracted and replaced by a step in the opposite direction, which in turn is retained depending upon whether it succeeds or fails. When all N coordinates have been investigated, the exploratory move is completed. The resulting point is termed a base point.

Hook and Jeeves (Pattern Search) Method

Pattern Move

A pattern move consists of a single step from the present base point along the line from the previous to the current base point. Thus, a new pattern point is calculated as;

$x^{(k)}$ Current base point
$x^{(k-1)}$ Previous base point
$x_p^{(k+1)}$ Pattern move point
$x^{(k+1)}$ Next (or new) base poi

$$x_p^{(k+1)} = x^{(k)} + (x^{(k)} - x^{(k-1)})$$

Now, recognizing that this move may not result in an improvement, the point $x_p^{(k+1)}$ is accepted only temporarily. It becomes the temporary base point for a new exploratory move. If the result of this exploratory move is a better point than the previous base point $x^{(k)}$, then this point is accepted as the new base point $x^{(k+1)}$. On the other hand, if the exploratory move does not produce improvement, then the pattern move is discarded and the search returns to $x^{(k)}$, where an exploratory search is undertaken to find a new pattern. Eventually, a situation is reached when even this exploratory search fails. In this case the step sizes are reduced by some factor and the exploration resumed. The search is terminated when the step size becomes sufficiently small.

Hook and Jeeves (Pattern Search) Method

Step 1. Define:

The starting point $x^{(0)}$

The increments Δ_i for $k = 1, 2, 3, \ldots, N$

The step reduction factor $\alpha > 1$

A termination parameter $\varepsilon > 0$

Step 2. Perform exploratory search.

Step 3. Was exploratory search successful (i.e., was a lower point found)?

Yes: Go to 5.

No: Continue.

Step 4. Check for termination.

Is $\|\Delta\| < \varepsilon$?

Yes: Stop; current point approximates x^*.

No: Reduce the increments:

$$\Delta_i = \frac{\Delta_i}{\alpha} \qquad i = 1, 2, 3, \ldots, N$$

Go to 2.

Hook and Jeeves (Pattern Search) Method

Step 5. Perform pattern move:

$$x_p^{(k+1)} = x^{(k)} + (x^{(k)} - x^{(k-1)})$$

Step 6. Perform exploratory research using $x_p^{(k+1)}$ as the base point; let the result be $x^{(k+1)}$.

Step 7. Is $f(x^{(k+1)}) < f(x^{(k)})$?

Yes: Set $x^{(k-1)} = x^{(k)}$; $x^{(k)} = {}^{(k+1)}$.

Go to 5.

No: Go to 4.

Hook and Jeeves (Pattern Search) Method - Example

Find the minimum of

$$f(x) = 8x_1^2 + 4x_1x_2 + 5x_2^2$$

from $x^{(0)} = [-4, -4]^T$.

Solution

To use the HJ direct-search method

$$\Delta = \text{step-size increments} = [1, 1]^T$$

$$\alpha = \text{step reduction factor} = 2$$

$$\varepsilon = \text{termination parameter} = 10^{-4}$$

Begin the iteration with an exploration about $x^{(0)}$, with $f(x^{(0)}) = 272$. With x_2 fixed, we increment x_1: With $x_2 = -4$

Hook and Jeeves (Pattern Search) Method - Example

$$x_1 = -4 + 1 \rightarrow f(-3, -4) = 200 < f(\bar{x}^{(0)}) \rightarrow \text{success}$$

Therefore, we fix x_1 at -3 and increment x_2: With $x_1 = -3$

$$x_2 = -4 + 1 \rightarrow f(-3, -3) = 153 < 20 \rightarrow \text{success}$$

Therefore the result of the first exploration is

$$x^{(1)} = [-3, -3]^T \qquad f(x^{(1)}) = 153$$

Since the exploration was a success, we go directly to the pattern move:

$$x_p^2 = x^{(1)} + (x^{(1)} - x^{(0)}) = [-2, -2]^T$$

$$f(x_p^{(2)}) = 68$$

Hook and Jeeves (Pattern Search) Method - Example

$$f(x_p^{(2)}) = 68$$

Now we perform an exploratory search about $x_p^{(2)}$. We find that positive increments on x_1 and x_2 produce success. The result is the point

$$x^{(2)} = [-1, -1]^T \qquad f(x^{(2)}) = 17$$

Since $f(x^{(2)}) < f(x^{(1)})$, the pattern move is deemed a success and $x^{(2)}$ becomes the new base point for an additional pattern move. The method continues until step reduction causes termination near the solution, $x^* = [0, 0]^T$.

Hook and Jeeves (Pattern Search) Method - Example