Introduction to Engineering Optimization (ME6806)

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Bisection Method

Determine two points L and R such that $f'(L) < 0$ and $f'(R) > 0$. The stationary point is between the points L and R. We determine the derivative of the function at the midpoint,

$$z = \frac{L + R}{2}$$

If $f'(z) > 0$, then the interval (z, R) can be eliminated from the search. On the other hand, if $f'(z) < 0$, then the interval (L, z) can be eliminated. We shall now state the formal steps of the algorithm.

Given a bounded interval $a \leqslant x \leqslant b$ and a termination criterion ε:

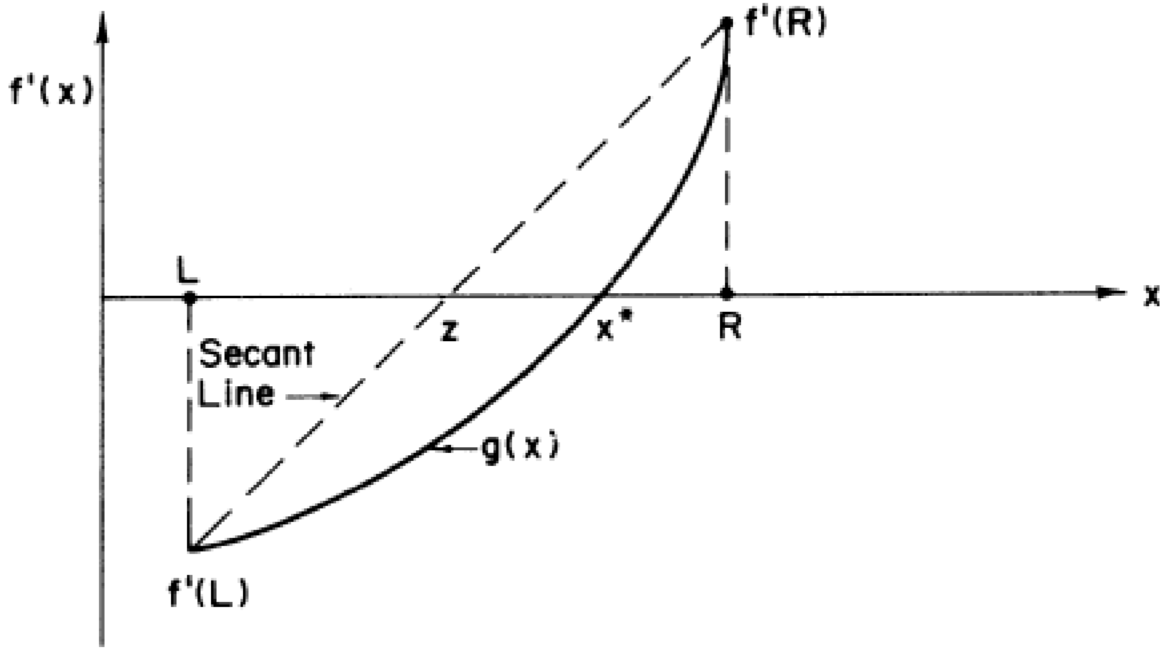
Step 1. Set $R = b$, $L = a$; assume $f'(a) < 0$ and $f'(b) > 0$.

Step 2. Calculate $z = (R + L)/2$, and evaluate $f'(z)$.

Step 3. If $|f'(z)| \leqslant \varepsilon$, terminate. Otherwise, if $f'(z) < 0$, set $L = z$ and go to step 2. If $f'(z) > 0$, set $R = z$ and go to step 2.

Secant Method

$$z = R - \frac{f'(R)}{[f'(R) - f'(L)]/(R - L)}$$



Secant Method

Consider again the problem

$$\text{Minimize } f(x) = 2x^2 + \frac{16}{x} \text{ over the interval } 1 \leqslant x \leqslant 5$$

$$f'(x) = \frac{df(x)}{dx} = 4x - \frac{16}{x^2}$$

$$z = R - \frac{f'(R)}{[f'(R) - f'(L)]/(R - L)}$$

Secant Method - Example

Iteration 1

Step 1. $R = 5 \qquad L = 1 \qquad f'(R) = 19.36 \qquad f'(L) = -12$

Step 2. $z = 5 - \dfrac{19.36}{(19.36 + 12)/4} = 2.53$

Step 3. $f'(z) = 7.62 > 0$; set $R = 2.53$.

Iteration 2

Step 2. $z = 2.53 - \dfrac{7.62}{(7.62 + 12)/1.53} = 1.94$

Step 3. $f'(z) = 3.51 > 0$; set $R = 1.94$.

Continue until $|f'(z)| \leqslant \varepsilon$.

Cubic Search Method

$$\bar{f}(x) = a_0 + a_1(x - x_1) + a_2(x - x_1)(x - x_2) + a_3(x - x_1)^2(x - x_2)$$

$$\frac{d\bar{f}(x)}{dx} = a_1 + a_2(x - x_1) + a_2(x - x_2) + a_3(x - x_1)^2 + 2a_3(x - x_1)(x - x_2)$$

$$f_1 = f(x_1) = a_0$$

$$f_2 = f(x_2) = a_0 + a_1(x_2 - x_1)$$

$$f_1' = f'(x_1) = a_1 + a_2(x_1 - x_2)$$

$$f_2' = f'(x_2) = a_1 + a_2(x_2 - x_1) + a_3(x_2 - x_1)^2$$

Cubic Search Method

Given an initial point x_0, positive step size Δ, and termination parameters ε_1 and ε_2, the formal steps of the cubic search algorithm are as follows:

Step 1. Compute $f'(x_0)$.

If $f'(x_0) < 0$, compute $x_{K+1} = x_K + 2^K\Delta$ for $K = 0, 1, \ldots$.

If $f'(x_0) > 0$, compute $x_{K+1} = x_K - 2^K\Delta$ for $K = 0, 1, 2, \ldots$.

Step 2. Evaluate $f'(x)$ for points x_{K+1} for $K = 0, 1, 2, \ldots$ until a point x_M is reached at which $f'(x_{M-1})f'(x_M) \leqslant 0$.

Then set $x_1 = x_{M-1}$, $x_2 = x_M$.

Compute f_1, f_2, f'_1, and f'_2.

Step 3. Calculate the stationary point $\bar{x}$ of the cubic approximating function using Eq. (2.10).

Step 4. If $f(\bar{x}) < f(x_1)$, go to step 5. Otherwise, set $\bar{x} = \bar{x} + \frac{1}{2}(\bar{x} - x_1)$ until $f(\bar{x}) \leqslant f(x_1)$ is achieved.

Step 5. Termination check:

If $|f'(\bar{x})| \leqslant \varepsilon_1$ and $|(\bar{x} - x_1)/\bar{x}| \leqslant \varepsilon_2$, stop. Otherwise, set

(i) $x_2 = x_1$ and $x_1 = \bar{x}$ if $f'(\bar{x})f'(x_1) < 0$

(ii) $x_1 = \bar{x}$ if $f'(\bar{x})f'(x_2) < 0$

In either case, continue with step 3.