Introduction to Engineering Optimization (ME6806)

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Successive Quadratic Estimation Method

Step 1. Compute $x_2 = x_1 + \Delta x$.

Step 2. Evaluate $f(x_1)$ and $f(x_2)$.

Step 3. If $f(x_1) > f(x_2)$, let $x_3 = x_1 + 2\Delta x$.
If $f(x_1) \leqslant f(x_2)$, let $x_3 = x_1 - \Delta x$.

Step 4. Evaluate $f(x_3)$ and determine

$$F_{\min} = \min\{f_1, f_2, f_3\}$$

$$X_{\min} = \text{point } x_i \text{ corresponding to } F_{\min}$$

Successive Quadratic Estimation Method

Step 5. Use the points x_1, x_2, x_3 to calculate $\bar{x}$ using the quadratic estimation formula.

Step 6. Check for termination.

(a) Is $F_{min} - f(\bar{x})$ small enough?

(b) Is $X_{min} - \bar{x}$ small enough?

If both are satisfied, terminate. Otherwise, go to step 7.

Step 7. Save the currently best point (X_{min} or $\bar{x}$), the two points bracketing it, or the two closest to it. Relabel them and go to step 4.

Successive Quadratic Estimation Method - Example

Consider the problem

$$\text{Minimize } f(x) = 2x^2 + \frac{16}{x}$$

with initial point $x_1 = 1$ and step size $\Delta x = 1$. For convergence parameters use

$$\left|\frac{\text{Difference in } x}{x}\right| \leqslant 3 \times 10^{-2} \qquad \left|\frac{\text{Difference in } F}{F}\right| \leqslant 3 \times 10^{-3}$$

Iteration 1

Step 1. $x_2 = x_1 + \Delta x = 2$

Step 2. $f(x_1) = 18 \qquad f(x_2) = 16$

Step 3. $f(x_1) > f(x_2)$; therefore set $x_3 = 1 + 2 = 3$.

Successive Quadratic Estimation Method - Example

Step 4. $f(x_3) = 23.33$

$$F_{\min} = 16$$

$$X_{\min} = x_2$$

Step 5. $a_1 = \dfrac{16 - 18}{2 - 1} = -2$

$$a_2 = \frac{1}{3 - 2}\left(\frac{23.33 - 18}{3 - 1} - a_1\right) = \frac{5.33}{2} + 2 = 4.665$$

$$\bar{x} = \frac{1 + 2}{2} - \frac{-2}{2(4.665)} = 1.5 + \frac{1}{4.665} = 1.714$$

$$f(\bar{x}) = 15.210$$

Step 6. Test for termination:

$$\left|\frac{16 - 15.210}{15.210}\right| = 0.0519 > 0.003$$

Successive Quadratic Estimation Method - Example

Step 7. Save $\bar{x}$, the currently best point, and x_1 and x_2, the two points that bound it. Relabel the points in order and go to iteration 2, starting with step 4.

Iteration 2

Step 4.
$$x_1 = 1 \qquad f_1 = 18$$
$$x_2 = 1.714 \qquad f_2 = 15.210 = F_{\min} \qquad \text{and} \qquad X_{\min} = x_2$$
$$x_3 = 2 \qquad f_3 = 16$$

Step 5.
$$a_1 = \frac{15.210 - 18}{1.714 - 1} = -3.908$$
$$a_2 = \frac{1}{2 - 1.714}\left(\frac{16 - 18}{2 - 1} - (-3.908)\right) = \frac{1.908}{0.286} = 6.671$$
$$\bar{x} = \frac{2.714}{2} - \frac{-3.908}{2(6.671)} = 1.357 + 0.293 = 1.650$$
$$f(\bar{x}) = 15.142$$

Successive Quadratic Estimation Method - Example

Step 6. Test for termination:

$$\left|\frac{15.210 - 15.142}{15.142}\right| = 0.0045 > 0.003 \qquad \text{not satisfied}$$

Step 7. Save $\bar{x}$, the currently best point, and $x_1 = 1$ and $x_2 = 1.714$, the two points that bracket it.

Iteration 3

Step 4.

$$x_1 = 1 \qquad f_1 = 18$$
$$x_2 = 1.65 \qquad f_2 = 15.142 = F_{\min} \qquad \text{and} \qquad X_{\min} = x_2$$
$$x_3 = 1.714 \qquad f_3 = 15.210$$

Successive Quadratic Estimation Method - Example

Step 5. $a_1 = \dfrac{15.142 - 18}{1.65 - 1} = -4.397$

$$a_2 = \frac{1}{1.714 - 1.650}\left(\frac{15.210 - 18}{1.714 - 1} - (-4.397)\right) = 7.647$$

$$\bar{x} = \frac{2.65}{2} - \frac{-4.397}{2(7.647)} = 1.6125$$

$$f(\bar{x}) = 15.123$$

Step 6. Test for termination:

(i) $\left|\dfrac{15.142 - 15.123}{15.123}\right| = 0.0013 < .003$

(ii) $\left|\dfrac{1.65 - 1.6125}{1.6125}\right| = 0.023 < 0.03$

Therefore, terminate iterations.