Introduction to Engineering Optimization (ME6806)

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Module 3

Single-variable Optimization Algorithms

Polynomial Approximation or Point - Estimation Technique

- Quadratic Approximation Method

Given three consecutive points x_1, x_2, x_3 and their corresponding function values f_1, f_2, f_3, we seek to determine three constants a_0, a_1, and a_2 such that the quadratic function

$$q(x) = a_0 + a_1(x - x_1) + a_2(x - x_1)(x - x_2)$$

agrees with $f(x)$ at these three points. We proceed by evaluating $q(x)$ at each of the three given points. First of all, since

$$f_1 = f(x_1) = q(x_1) = a_0$$

we have

$$a_0 = f_1$$

Next, since

$$f_2 = f(x_2) = q(x_2) = f_1 + a_1(x_2 - x_1)$$

we have

$$a_1 = \frac{f_2 - f_1}{x_2 - x_1}$$

Finally, at $x = x_3$,

$$f_3 = f(x_3) = q(x_3) = f_1 + \frac{f_2 - f_1}{x_2 - x_1}(x_3 - x_1) + a_2(x_3 - x_1)(x_3 - x_2)$$

Solving for a_2, we obtain

$$a_2 = \frac{1}{x_3 - x_2}\left(\frac{f_3 - f_1}{x_3 - x_1} - \frac{f_2 - f_1}{x_2 - x_1}\right)$$

In the case of our quadratic approximating function,

$$\frac{dq}{dx} = a_1 + a_2(x - x_2) + a_2(x - x_1) = 0$$

can be solved to yield the estimate

$$\bar{x} = \frac{x_2 + x_1}{2} - \frac{a_1}{2a_2}$$

Quadratic Search - Example

Consider the estimation of the minimum of

$$f(x) = 2x^2 + \frac{16}{x}$$

on the interval $1 \leqslant x \leqslant 5$. Let $x_1 = 1$, $x_3 = 5$, and choose as x_2 the midpoint $x_2 = 3$. Evaluating the function, we obtain

$$f_1 = 18 \qquad f_2 = 23.33 \qquad f_3 = 53.2$$

Quadratic Search - Example

To calculate the estimate $\bar{x}$, the constants a_1 and a_2 of the approximating function must be evaluated. Thus,

$$a_1 = \frac{23.33 - 18}{3 - 1} = \frac{8}{3}$$

$$a_1 = \frac{f_2 - f_1}{x_2 - x_1}$$

$$a_2 = \frac{1}{5 - 3}\left(\frac{53.2 - 18}{5 - 1} - \frac{8}{3}\right) = \frac{46}{15}$$

$$a_2 = \frac{1}{x_3 - x_2}\left(\frac{f_3 - f_1}{x_3 - x_1} - \frac{f_2 - f_1}{x_2 - x_1}\right)$$

Substituting into the expression for $\bar{x}$,

$$\bar{x} = \frac{3 + 1}{2} - \frac{8/3}{2(46/15)} = 1.565$$

$$\bar{x} = \frac{x_2 + x_1}{2} - \frac{a_1}{2a_2}$$

The exact minimum is $x^* = 1.5874$.