Introduction to Engineering Optimization (ME6806)

Dr. Yogesh Kumar
Assistant Professor
Mechanical Engineering Department
National Institute of Technology Patna
Bihar - 800 005, India
yogesh.me@nitp.ac.in

Module 3

Single-variable Optimization Algorithms

Interval Refinement Phase - Interval halving

Interval Halving. This method deletes *exactly* one-half the interval at each stage. This is also called a *three-point equal-interval search* since it works with three equally spaced trial points in the search interval. The basic steps

Step 1. Let $x_m = \frac{1}{2}(a + b)$ and $L = b - a$. Compute $f(x_m)$.

Step 2. Set $x_1 = a + \frac{1}{4}L$ and $x_2 = b - \frac{1}{4}L$.

Note that the points x_1, x_m, and x_2 are all equally spaced at one-fourth the interval. Compute $f(x_1)$ and $f(x_2)$.

Step 3. Compare $f(x_1)$ and $f(x_m)$.

(i) If $f(x_1) < f(x_m)$, then drop the interval (x_m, b) by setting $b = x_m$. The midpoint of the new search interval will now be x_1. Hence, set $x_m = x_1$. Go to step 5.

(ii) If $f(x_1) \geqslant f(x_m)$, go to step 4.

Interval Refinement Phase - Interval halving

Step 4. Compare $f(x_2)$ and $f(x_m)$.

(i) If $f(x_2) < f(x_m)$, drop the interval (a, x_m) by setting $a = x_m$. Since the midpoint of the new interval will now be x_2, set $x_m = x_2$. Go to step 5.

(ii) If $f(x_2) \geqslant f(x_m)$, drop the interval (a, x_1) and (x_2, b). Set $a = x_1$ and $b = x_2$. Note that x_m continues to be the midpoint of the new interval. Go to step 5.

Step 5. Compute $L = b - a$. If $|L|$ is small, terminate. Otherwise return to step 2.

Remarks

1. At each stage of the algorithm, exactly half the length of the search interval is deleted.
2. The midpoint of subsequent intervals is always equal to one of the previous trial points x_1, x_2, or x_m. Hence, at most two functional evaluations are necessary at each subsequent step.
3. After n functional evaluations, the initial search interval will be reduced to $(\frac{1}{2})^{n/2}$.

Interval halving – Example

Minimize $f(x) = (100 - x)^2$ over the interval $60 \leqslant x \leqslant 150$. Here $a = 60$, $b = 150$, and $L = 150 - 60 = 90$.

$$x_m = \tfrac{1}{2}(60 + 150) = 105$$

Stage 1

$$x_1 = a + \tfrac{1}{4}L = 60 + \tfrac{90}{4} = 82.5$$

$$x_2 = b - \tfrac{1}{4}L = 150 - \tfrac{90}{4} = 127.5$$

$$f(82.5) = 306.25 > f(105) = 25$$

$$f(127.5) = 756.25 > f(105)$$

Hence, drop the intervals (60, 82.5) and (127.5, 150). The length of the search interval is reduced from 90 to 45.

Interval halving – Example

Stage 2

$$a = 82.5 \qquad b = 127.5 \qquad x_m = 105$$

$$L = 127.5 - 82.5 = 45$$

$$x_1 = 82.5 + \tfrac{45}{4} = 93.75$$

$$x_2 = 127.5 - \tfrac{45}{4} = 116.25$$

$$f(93.75) = 39.06 > f(105) = 25$$

$$f(116.25) = 264.06 > f(105)$$

Hence, the interval of uncertainty is (93.75, 116.25).

Interval halving – Example

Stage 3

$$a = 93.75 \qquad b = 116.25 \qquad x_m = 105$$

$$L = 116.25 - 93.75 = 22.5$$

$$x_1 = 99.375$$

$$x_2 = 110.625$$

$$f(x_1) = 0.39 < f(105) = 25$$

Hence, delete the interval (105, 116.25). The new interval of uncertainty is now (93.75, 105), and its midpoint is 99.375 (old x_1). Thus, in three stages (six functional evaluations), the initial search interval of length 90 has been reduced exactly to $(90)(\frac{1}{2})^3 = 11.25$.