Introduction to Engineering Optimization (ME6806)

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Module 2

Optimization Problems Using MATLAB

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- MATLAB is a popular software that is used for the solution of a variety of scientific and engineering problems.
- It has several toolboxes each developed for the solution of problems from a specific scientific area.
- The specific toolbox of interest for solving optimization and related problems is called the optimization toolbox
- optimization toolbox contains a library of programs or m-files, which can be used for the solution of minimization, equations, least squares curve fitting, and related problems.

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Optimization Problems Using MATLAB

- Step 1 involves writing an m-file for the objective function.
- Step 2 involves writing an m-file for the constraints.
- Step 3 involves setting the various parameters at proper values depending on the characteristics of the problem and the desired output and creating an appropriate file to invoke the desired MATLAB program (and coupling the m-files created to define the objective and constraints functions of the problem).

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Type of optimization problem	Standard form for solution by MATLAB	Name of MATLAB program or function to solve the problem
Function of one variable or scalar minimization	Find x to minimize $f(x)$ with $x_1 < x < x_2$	`fminbnd`
Unconstrained minimization of function of several variables	Find $\mathbf{x}$ to minimize $f(\mathbf{x})$	`fminunc` or `fminsearch`
Linear programming problem	Find $\mathbf{x}$ to minimize $\mathbf{f}^T\mathbf{x}$ subject to $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`linprog`
Quadratic programming problem	Find $\mathbf{x}$ to minimize $\frac{1}{2}\mathbf{x}^T[H]\mathbf{x} + \mathbf{f}^T\mathbf{x}$ subject to $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`quadprog`
Minimization of function of several variables subject to constraints	Find $\mathbf{x}$ to minimize $f(\mathbf{x})$ subject to $\mathbf{c}(\mathbf{x}) \le \mathbf{0}$, $\mathbf{c}_{\text{eq}} = \mathbf{0}$ $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`fmincon`
Goal attainment problem	Find $\mathbf{x}$ and γ to minimize γ such that $F(\mathbf{x}) - \mathbf{w}\gamma \le \mathbf{goal}$, $\mathbf{c}(\mathbf{x}) \le \mathbf{0}$, $\mathbf{c}_{\text{eq}} = \mathbf{0}$ $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`fgoalattain`
Minimax problem	$\underset{\mathbf{x}}{\text{Minimize}} \ \underset{[F_i]}{\text{Max}} \ [F_i(\mathbf{x})]$ such that $\mathbf{c}(\mathbf{x}) \le \mathbf{0}$, $\mathbf{c}_{\text{eq}} = \mathbf{0}$ $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, $\mathbf{l} \le \mathbf{x} \le \mathbf{u}$	`fminimax`
Binary integer programming problem	Find $\mathbf{x}$ to minimize $\mathbf{f}^T\mathbf{x}$ subject to $[A]\mathbf{x} \le \mathbf{b}$, $[A_{\text{eq}}]\mathbf{x} = \mathbf{b}_{\text{eq}}$, each component of $\mathbf{x}$ is binary	`bintprog`

Example 1

Design a uniform column of tubular section to carry a compressive load $P = 2500$ *kgf* for minimum cost. The column is made up of a material that has a yield stress of 500 kgf/cm^2, modulus of elasticity (E) of $0.85x10^6 kgf/cm^2$, and density (ρ) of 0.0025 kgf/cm^3. The length of the column is 250 *cm*. The stress induced in this column should be less than the buckling stress as well as the yield stress. The mean diameter of the column is restricted to lie between 2 and 14 *cm*, and columns with thicknesses outside the range 0.2 to 0.8 *cm* are not available in the market. The cost of the column includes material and construction costs and can be taken as $5W + 2d$, where W is the weight in kilograms force and d is the mean diameter of the column in centimeters.

Example 1

The design variables are the mean diameter (d) and tube thickness (t):

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} d \\ t \end{Bmatrix}$$

The objective function to be minimized is given by:

$$f(\mathbf{X}) = 5W + 2d = 5\rho\pi dt + 2d = 9.82x_1x_2 + 2x_1$$

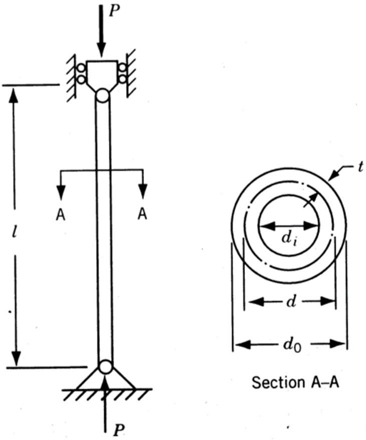

Figure: Tubular column under compression

Example 1

The behavior constraints can be expressed as;

$$\text{stress induced} \leq \text{yield stress}$$
$$\text{stress induced} \leq \text{buckling stress}$$

The induced stress is given by;

$$\text{induced stress} = \sigma_i = \frac{P}{\pi dt} = \frac{2500}{\pi x_1 x_2}$$

The buckling stress for a pin-connected column is given by;

$$\text{buckling stress} = \sigma_b = \frac{Euler\ buckling\ load}{cross-sectional\ area} = \frac{\pi^2 EI}{l^2}\frac{1}{\pi dt}$$

Example 1

Where,

I = second moment of area of the cross section of the column

$$= \frac{\pi}{64}(d_o^4 - d_i^4)$$

$$= \frac{\pi}{64}(d_o^2 + d_i^2)(d_o + d_i)(d_o - d_i) = \frac{\pi}{64}[(d+t)^2 + (d-t)^2]$$

$$\times [(d+t) + (d-t)][(d+t) - (d-t)]$$

$$= \frac{\pi}{8} dt(d^2 + t^2) = \frac{\pi}{8} x_1 x_2 (x_1^2 + x_2^2)$$

Thus the behavior constraints can be restated as

$$g_1(\mathbf{X}) = \frac{2500}{\pi x_1 x_2} - 500 \le 0$$

$$g_2(\mathbf{X}) = \frac{2500}{\pi x_1 x_2} - \frac{\pi^2 (0.85 \times 10^6)(x_1^2 + x_2^2)}{8(250)^2} \le 0$$

Example 1

The side constraints are given by

$$2 \leq d \leq 14$$
$$0.2 \leq t \leq 0.8$$

which can be expressed in standard form as

$$g_3(\mathbf{X}) = -x_1 + 2.0 \leq 0$$
$$g_4(\mathbf{X}) = x_1 - 14.0 \leq 0$$
$$g_5(\mathbf{X}) = -x_2 + 0.2 \leq 0$$
$$g_6(\mathbf{X}) = x_2 - 0.8 \leq 0$$

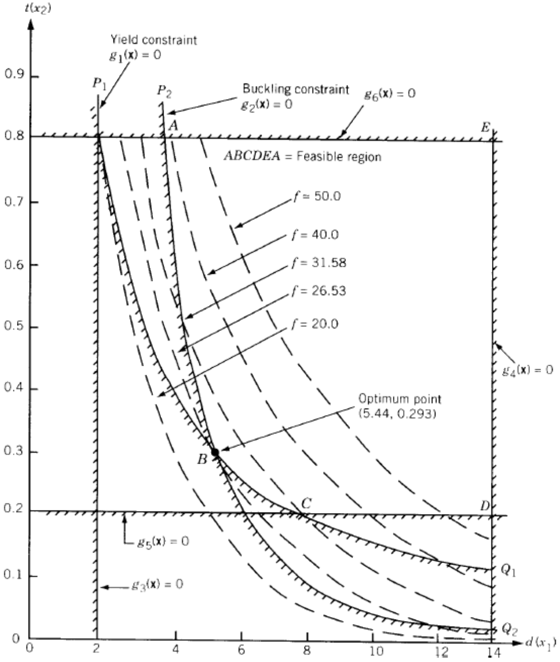

Figure 1 Graphical optimization