Introduction to Engineering Optimization (ME6806)

Dr. Yogesh Kumar
Assistant Professor
Mechanical Engineering Department
National Instiute of Technology Patna
Bihar - 800 005, India
yogesh.me@nitp.ac.in

Module 2

Lecture Date 12.01.2021

Example 3

A manufacturing firm produces two products, A and B, using two limited resources. The maximum amounts of resources 1 and 2 available per day are 1000 and 250 units, respectively. The production of 1 unit of product A requires 1 unit of resource 1 and 0.2 unit of resource 2, and the production of 1 unit of product B requires 0.5 unit of resource 1 and 0.5 unit of resource 2. The unit costs of resources 1 and 2 are given by the relations $(0.375 - 0.00005u_1)$ and $(0.75 - 0.0001u_2)$, respectively, where u_i denotes the number of units of resource i used (i = 1, 2). The selling prices per unit of products A and B, p_A and p_B, are given by

$$p_A = 2.00 - 0.0005x_A - 0.00015x_B$$

$$p_B = 3.50 - 0.0002x_A - 0.0015x_B$$

where x_A and x_B indicate, respectively, the number of units of products A and B sold. Formulate the problem of maximizing the profit assuming that the firm can sell all the units it manufactures.

Example 3

Let the design variables be the number of units of products A and B manufactured per day:

$$\mathbf{X} = \begin{Bmatrix} x_A \\ x_B \end{Bmatrix}$$

The requirement of resource 1 per day is $(x_A + 0.5x_B)$ and that of resource 2 is $(0.2x_A + 0.5x_B)$ and the constraints on the resources are

$$x_A + 0.5x_B \leq 1000 \qquad (E_1)$$

$$0.2x_A + 0.5x_B \leq 250 \qquad (E_2)$$

The lower bounds on the design variables can be taken as

$$x_A \geq 0 \qquad (E_3)$$

$$x_B \geq 0 \qquad (E_4)$$

The total cost of resources 1 and 2 per day is

$$(x_A + 0.5x_B)[0.375 - 0.00005(x_A + 0.5x_B)]$$
$$+ (0.2x_A + 0.5x_B)[0.750 - 0.0001(0.2x_A + 0.5x_B)]$$

Example 3

and the return per day from the sale of products A and B is

$$x_A(2.00 - 0.0005x_A - 0.00015x_B) + x_B(3.50 - 0.0002x_A - 0.0015x_B)$$

The total profit is given by the total return minus the total cost. Since the objective function to be minimized is the negative of the profit per day, $f(\mathbf{X})$ is given by

$$\begin{aligned} f(\mathbf{X}) = {} & (x_A + 0.5x_B)[0.375 - 0.00005(x_A + 0.5x_B)] \\ & + (0.2x_A + 0.5x_B)[0.750 - 0.0001(0.2x_A + 0.5x_B)] \\ & - x_A(2.00 - 0.0005x_A - 0.00015x_B) \\ & - x_B(3.50 - 0.0002x_A - 0.0015x_B) \end{aligned} \qquad (E_5)$$

As the objective function [Eq. (E_5)] is a quadratic and the constraints [Eqs. (E_1) to (E_4)] are linear, the problem is a quadratic programming problem.

Example 4

A scaffolding system consists of three beams and six ropes as shown in Fig. Each of the top ropes A and B can carry a load of W_1, each of the middle ropes C and D can carry a load of W_2, and each of the bottom ropes E and F can carry a load of W_3. If the loads acting on beams 1, 2, and 3 are x_1, x_2, and x_3, respectively, formulate the problem of finding the maximum

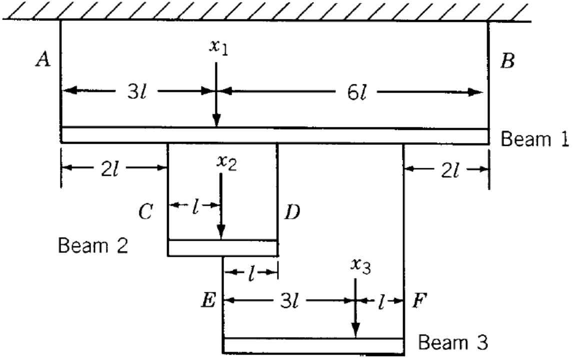

Figure Scaffolding system with three beams.

Example 4

load $(x_1 + x_2 + x_3)$ that can be supported by the system. Assume that the weights of the beams 1, 2, and 3 are w_1, w_2, and w_3, respectively, and the weights of the ropes are negligible.

SOLUTION Assuming that the weights of the beams act through their respective middle points, the equations of equilibrium for vertical forces and moments for each of the three beams can be written as

For beam 3:

$$T_E + T_F = x_3 + w_3$$

$$x_3(3l) + w_3(2l) - T_F(4l) = 0$$

For beam 2:

$$T_C + T_D - T_E = x_2 + w_2$$

$$x_2(l) + w_2(l) + T_E(l) - T_D(2l) = 0$$

For beam 1:

$$T_A + T_B - T_C - T_D - T_F = x_1 + w_1$$

$$x_1(3l) + w_1(\tfrac{9}{2}l) - T_B(9l) + T_C(2l) + T_D(4l) + T_F(7l) = 0$$

Example 4

where T_i denotes the tension in rope i. The solution of these equations gives

$$\begin{aligned}
T_F &= \tfrac{3}{4}x_3 + \tfrac{1}{2}w_3 \\
T_E &= \tfrac{1}{4}x_3 + \tfrac{1}{2}w_3 \\
T_D &= \tfrac{1}{2}x_2 + \tfrac{1}{8}x_3 + \tfrac{1}{2}w_2 + \tfrac{1}{4}w_3 \\
T_C &= \tfrac{1}{2}x_2 + \tfrac{1}{8}x_3 + \tfrac{1}{2}w_2 + \tfrac{1}{4}w_3 \\
T_B &= \tfrac{1}{3}x_1 + \tfrac{1}{3}x_2 + \tfrac{2}{3}x_3 + \tfrac{1}{2}w_1 + \tfrac{1}{3}w_2 + \tfrac{5}{9}w_3 \\
T_A &= \tfrac{2}{3}x_1 + \tfrac{2}{3}x_2 + \tfrac{1}{3}x_3 + \tfrac{1}{2}w_1 + \tfrac{2}{3}w_2 + \tfrac{4}{9}w_3
\end{aligned}$$

The optimization problem can be formulated by choosing the design vector as

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}$$

Since the objective is to maximize the total load

$$f(\mathbf{X}) = -(x_1 + x_2 + x_3) \qquad (E_1)$$

Example 4

The constraints on the forces in the ropes can be stated as

$$T_A \leq W_1 \tag{E$_2$}$$

$$T_B \leq W_1 \tag{E$_3$}$$

$$T_C \leq W_2 \tag{E$_4$}$$

$$T_D \leq W_2 \tag{E$_5$}$$

$$T_E \leq W_3 \tag{E$_6$}$$

$$T_F \leq W_3 \tag{E$_7$}$$

Finally, the nonnegativity requirement of the design variables can be expressed as

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$x_3 \geq 0 \tag{E$_8$}$$

Since all the equations of the problem (E_1) to (E_8), are linear functions of x_1, x_2, and x_3, the problem is a linear programming problem.