Introduction to Engineering Optimization (ME6806)

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Module 2

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Classification Based on the Physical Structure of the Problem

- Optimal Control Problem: is a mathematical programming problem involving a number of stages, where each stage evolves from the preceding stage in a prescribed manner. It is usually described by two types of variables: the control (design) and the state variables. The control variables define the system and govern the evolution of the system from one stage to the next, and the state variables describe the behavior or status of the system in any stage. The problem is to find a set of control or design variables such that the total objective function over all the stages is minimized subject to a set of constraints on the control and state variables.
- Non-Optimal Control Problem: The problems which are not optimal control problems are called non-optimal control problems.

Example 2

A rocket is designed to travel a distance of *12s* in a vertically upward direction. The thrust of the rocket can be changed only at the discrete points located at distances of *0, s, 2s, 3s, . . . , 12s*. If the maximum thrust that can be developed at point *i* either in the positive or negative direction is restricted to a value of F_i , formulate the problem of minimizing the total time of travel under the following assumptions:

1. The rocket travels against the gravitational force.
2. The mass of the rocket reduces in proportion to the distance traveled.
3. The air resistance is proportional to the velocity of the rocket.

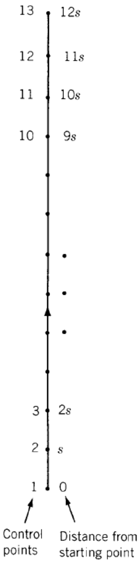
Example 2 | Optimal Control Problem

Let points (or control points) on the path at which the thrusts of the rocket are changed be numbered as 1, 2, 3, . . . , 13 (As shown in Fig.). Denoting x_i as the thrust, v_i the velocity, a_i the acceleration, and m_i the mass of the rocket at point i, Newton's second law of motion can be applied as

net force on the rocket $=$ *mass x acceleration*

This can be written as;

thrust $-$ *gravitational force* $-$ *air resistance* $=$ *mass x acceleration*



Example 2

$$x_i - m_i g - k_1 v_i = m_i a_i \tag{E$_1$}$$

where the mass m_i can be expressed as

$$m_i = m_{i-1} - k_2 s \tag{E$_2$}$$

and k_1 and k_2 are constants. Equation (E$_1$) can be used to express the acceleration, a_i, as

$$a_i = \frac{x_i}{m_i} - g - \frac{k_1 v_i}{m_i} \tag{E$_3$}$$

If t_i denotes the time taken by the rocket to travel from point i to point $i+1$, the distance traveled between the points i and $i+1$ can be expressed as

$$s = v_i t_i + \tfrac{1}{2} a_i t_i^2$$

or

$$\frac{1}{2} t_i^2 \left(\frac{x_i}{m_i} - g - \frac{k_1 v_i}{m_i} \right) + t_i v_i - s = 0 \tag{E$_4$}$$

Example 2

from which t_i can be determined as

$$t_i = \frac{-v_i \pm \sqrt{v_i^2 + 2s\left(\dfrac{x_i}{m_i} - g - \dfrac{k_1 v_i}{m_i}\right)}}{\dfrac{x_i}{m_i} - g - \dfrac{k_1 v_i}{m_i}} \tag{E$_5$}$$

Of the two values given by Eq. (E$_5$), the positive value has to be chosen for t_i. The velocity of the rocket at point $i+1$, v_{i+1}, can be expressed in terms of v_i as (by assuming the acceleration between points i and $i+1$ to be constant for simplicity)

$$v_{i+1} = v_i + a_i t_i \tag{E$_6$}$$

The substitution of Eqs. (E$_3$) and (E$_5$) into Eq. (E$_6$) leads to

$$v_{i+1} = \sqrt{v_i^2 + 2s\left(\frac{x_i}{m_i} - g - \frac{k_1 v_i}{m_i}\right)} \tag{E$_7$}$$

From an analysis of the problem, the control variables can be identified as the thrusts, x_i, and the state variables as the velocities, v_i. Since the rocket starts at point 1 and stops at point 13,

$$v_1 = v_{13} = 0 \tag{E$_8$}$$

Example 2

Thus the problem can be stated as an OC problem as

$$\text{Find } \mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{12} \end{Bmatrix} \text{ which minimizes}$$

$$f(\mathbf{X}) = \sum_{i=1}^{12} t_i = \sum_{i=1}^{12} \left\{ \frac{-v_i + \sqrt{v_i^2 + 2s\left(\dfrac{x_i}{m_i} - g - \dfrac{k_1 v_i}{m_i}\right)}}{\dfrac{x_i}{m_i} - g - \dfrac{k_1 v_i}{m_i}} \right\}$$

subject to

$$m_{i+1} = m_i - k_2 s, \qquad i = 1, 2, \ldots, 12$$

$$v_{i+1} = \sqrt{v_i^2 + 2s\left(\frac{x_i}{m_i} - g - \frac{k_1 v_i}{m_i}\right)}, \qquad i = 1, 2, \ldots, 12$$

$$|x_i| \le F_i, \qquad i = 1, 2, \ldots, 12$$

$$v_1 = v_{13} = 0$$

Classification based on the nature of the equations involved

Based on the nature of expressions for the objective function and the constraints, optimization problems can be classified as linear, nonlinear, geometric and quadratic programming problems.

- Linear programming problem: the objective function and all the constraints are linear functions of the design variables.
- Nonlinear programming problem: the objectives and constraint functions is nonlinear.
- Geometric programming problem: the objective function and constraints are expressed as polynomials.
- Quadratic programming problem: quadratic objective function and linear constraints.

Classification based on the permissible values of the decision variables

Under this classification problems can be classified asintegerand real-valuedprogramming problems.

- Integer programming problem: If some or all of the design variables of an optimization problem are restricted to take only integer (or discrete) values, the problem is called an integer programming problem.
- Real-valued programming problem: A real-valued problem is that in which it is sought to minimize or maximize a real function by systematically choosing the values of real variables from within an allowed set. When the allowed set contains only real values, it is called a real-valued programming problem.

Classification based on deterministic nature of the variables

Under this classification, optimization problems can be classified as deterministicandstochastic programming problems.

- Deterministic programming problem: In this type of problems all the design variables are deterministic.
- Stochastic programming problem: In this type of an optimization problem some or all the parameters (design variables and/or pre-assigned parameters) are probabilistic (non deterministic or stochastic). For exampleestimates of life span of structures which have probabilistic inputs of the concrete strength and load capacity. A deterministic value of the life-span is non-attainable.

Classification based on separability of the functions

Based on the separability of the objective and constraint functions optimization problems can be classified as separableand non-separable programming problems.

- Separable programming problems: In this type of a problem the objective function and the constraints are separable..
- Non-Separable programming problems: In this type of a problem the objective function and the constraints are Non-separable..

Classification based on the number of objective functions

Under this classification objective functions can be classified as singleand multiobjectiveprogramming problems.

- Single-objective programming problem: in which there is only a single objective.
- Multi-objective programming problem: in which there are more than one objectives.