Introduction to Engineering Optimization (ME6806)

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Module 2

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Example 1

Design a uniform column of tubular section to carry a compressive load $P = 2500\ kgf$ for minimum cost. The column is made up of a material that has a yield stress of 500 kgf/cm^2, modulus of elasticity (E) of $0.85x10^6 kgf/cm^2$, and density (ρ) of 0.0025 kgf/cm^3. The length of the column is 250 cm. The stress induced in this column should be less than the buckling stress as well as the yield stress. The mean diameter of the column is restricted to lie between 2 and 14 cm, and columns with thicknesses outside the range 0.2 to 0.8 cm are not available in the market. The cost of the column includes material and construction costs and can be taken as $5W + 2d$, where W is the weight in kilograms force and d is the mean diameter of the column in centimeters.

Example 1

The design variables are the mean diameter (d) and tube thickness (t):

$$\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} d \\ t \end{Bmatrix}$$

The objective function to be minimized is given by:

$$f(\mathbf{X}) = 5W + 2d = 5\rho\pi dt + 2d = 9.82x_1x_2 + 2x_1$$

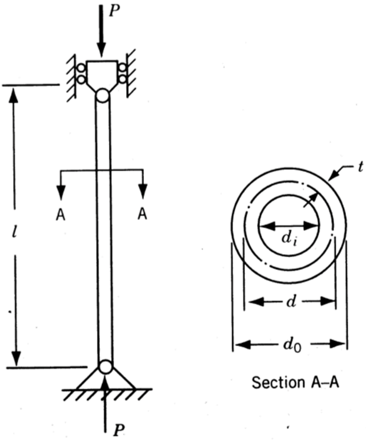


Figure: Tubular column under compression

Example 1

The behavior constraints can be expressed as;

$$\text{stress induced} \leq \text{yield stress}$$
$$\text{stress induced} \leq \text{buckling stress}$$

The induced stress is given by;

$$\text{induced stress} = \sigma_i = \frac{P}{\pi dt} = \frac{2500}{\pi x_1 x_2}$$

The buckling stress for a pin-connected column is given by;

$$\text{buckling stress} = \sigma_b = \frac{\textit{Euler buckling load}}{\textit{cross}-\textit{sectional area}} = \frac{\pi^2 EI}{l^2}\frac{1}{\pi dt}$$

Where,

I $=$ second moment of area of the cross section of the column

$$
\begin{aligned}
&= \frac{\pi}{64}(d_o^4 - d_i^4) \\
&= \frac{\pi}{64}(d_o^2 + d_i^2)(d_o + d_i)(d_o - d_i) = \frac{\pi}{64}[(d+t)^2 + (d-t)^2] \\
&\quad \times [(d+t) + (d-t)][(d+t) - (d-t)] \\
&= \frac{\pi}{8}\, dt\,(d^2 + t^2) = \frac{\pi}{8} x_1 x_2 (x_1^2 + x_2^2)
\end{aligned}
$$

Thus the behavior constraints can be restated as

$$g_1(\mathbf{X}) = \frac{2500}{\pi x_1 x_2} - 500 \le 0$$

$$g_2(\mathbf{X}) = \frac{2500}{\pi x_1 x_2} - \frac{\pi^2 (0.85 \times 10^6)(x_1^2 + x_2^2)}{8(250)^2} \le 0$$

Example 1

The side constraints are given by

$$2 \leq d \leq 14$$

$$0.2 \leq t \leq 0.8$$

which can be expressed in standard form as

$$g_3(\mathbf{X}) = -x_1 + 2.0 \leq 0$$

$$g_4(\mathbf{X}) = x_1 - 14.0 \leq 0$$

$$g_5(\mathbf{X}) = -x_2 + 0.2 \leq 0$$

$$g_6(\mathbf{X}) = x_2 - 0.8 \leq 0$$

Example 1

Since there are only two design variables, the problem can be solved graphically as shown below.

First, the constraint surfaces are to be plotted in a two-dimensional design space where the two axes represent the two design variables x_1 and x_2. To plot the first constraint surface, we have

$$g_1(\mathbf{X}) = \frac{2500}{\pi x_1 x_2} - 500 \leq 0$$

that is,

$$x_1 x_2 \geq 1.593$$

Thus the curve $x_1 x_2 = 1.593$ represents the constraint surface $g_1(\mathbf{X}) = 0$. This curve can be plotted by finding several points on the curve. The points on the curve can be found by giving a series of values to x_1 and finding the corresponding values of x_2 that satisfy the relation $x_1 x_2 = 1.593$:

x_1	2.0	4.0	6.0	8.0	10.0	12.0	14.0
x_2	0.7965	0.3983	0.2655	0.1990	0.1593	0.1328	0.1140

These points are plotted and a curve $P_1 Q_1$ passing through all these points is drawn as shown in Fig. 1. , and the infeasible region, represented by $g_1(\mathbf{X}) > 0$ or $x_1 x_2 < 1.593$, is shown by hatched lines.† Similarly, the second constraint $g_2(\mathbf{X}) \leq 0$ can be expressed as $x_1 x_2(x_1^2 + x_2^2) \geq 47.3$ and the points lying on the constraint surface $g_2(\mathbf{X}) = 0$ can be obtained as follows for $x_1 x_2(x_1^2 + x_2^2) = 47.3$:

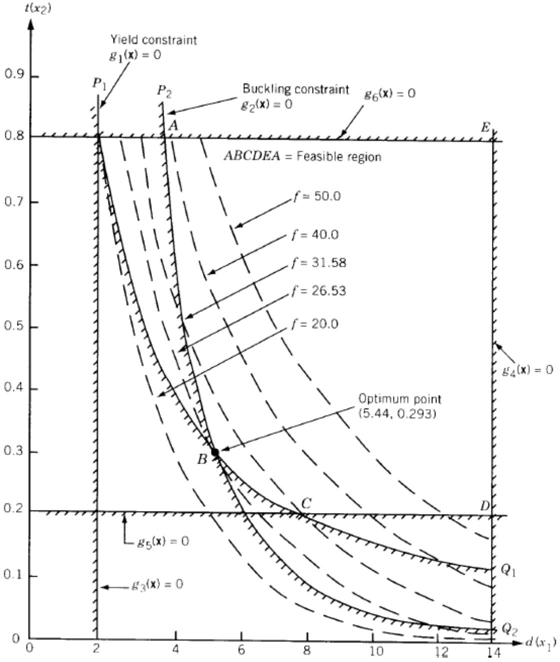


Figure 1 Graphical optimization

Example 1

x_1	2	4	6	8	10	12	14
x_2	2.41	0.716	0.219	0.0926	0.0473	0.0274	0.0172

These points are plotted as curve P_2Q_2, the feasible region is identified, and the infeasible region is shown by hatched lines as in Fig. 1 The plotting of side constraints is very simple since they represent straight lines. After plotting all the six constraints, the feasible region can be seen to be given by the bounded area *ABCDEA*.

Next, the contours of the objective function are to be plotted before finding the optimum point. For this, we plot the curves given by

$$f(\mathbf{X}) = 9.82x_1x_2 + 2x_1 = c = \text{constant}$$

for a series of values of c. By giving different values to c, the contours of f can be plotted with the help of the following points.

For $9.82x_1x_2 + 2x_1 = 50.0$:

x_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x_1	16.77	12.62	10.10	8.44	7.24	6.33	5.64	5.07

For $9.82x_1x_2 + 2x_1 = 40.0$:

x_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x_1	13.40	10.10	8.08	6.75	5.79	5.06	4.51	4.05

Example 1

For $9.82x_1x_2 + 2x_1 = 31.58$ (passing through the corner point C):

x_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x_1	10.57	7.96	6.38	5.33	4.57	4.00	3.56	3.20

For $9.82x_1x_2 + 2x_1 = 26.53$ (passing through the corner point B):

x_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x_1	8.88	6.69	5.36	4.48	3.84	3.36	2.99	2.69

For $9.82x_1x_2 + 2x_1 = 20.0$:

x_2	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
x_1	6.70	5.05	4.04	3.38	2.90	2.53	2.26	2.02

These contours are shown in Fig. 1.7 and it can be seen that the objective function cannot be reduced below a value of 26.53 (corresponding to point B) without violating some of the constraints. Thus the optimum solution is given by point B with $d^* = x_1^* = 5.44$ cm and $t^* = x_2^* = 0.293$ cm with $f_{\min} = 26.53$.

Classification of Engineering Problems

- Optimization problems can be classified based on the type of constraints, nature of design variables, physical structure of the problem, nature of the equations involved, deterministic nature of the variables, permissible value of the design variables, separability of the functions, and number of objective functions.

Classification Based on the Existence of Constraints

1. Constrained Optimization Problems

Which are subject to one or more constraints.

2. Unconstrained Optimization Problems

No constraints or Limitation exist.

Classification based on the nature of the design variables

- There are two broad categories of classification within this classification;
- First category : the objective is to find a set of design parameters that make a prescribed function of these parameters minimum or maximum subject to certain constraints.

Classification based on the nature of the design variables

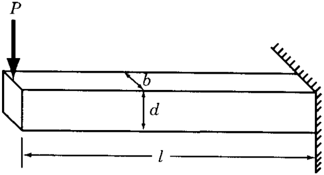
- the problem of minimum-weight design of a prismatic beam shown in Fig. subject to a limitation on the maximum deflection can be stated as follows:

Find $\mathbf{X} = \begin{Bmatrix} b \\ d \end{Bmatrix}$

which minimizes $f(\mathbf{X}) = \rho l b d$

Subject to the constraints

$$\delta_{tip}(\mathbf{X}) \leq \delta_{max}$$
$$b \geq 0$$
$$d \geq 0$$



(a)

Where ρ is the density and δ_{tip} is the tip deflection of the beam. Such problems are called *parameter* or *static optimization problems*.

Classification based on the nature of the design variables

- Second category : the objective is to find a set of design parameters, which are all continuous functions of some other parameter, that minimizes an objective function subject to a set of constraints.

$$\text{Find } \mathbf{X}(t) = \begin{Bmatrix} b(t) \\ d(t) \end{Bmatrix}$$

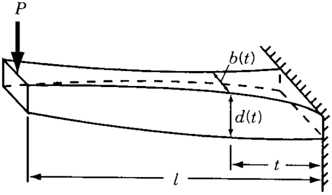
which minimizes $f[\mathbf{X}(t)] = \rho \int_0^l b(t) d(t)\, dt$

Subject to the constraints

$$\delta_{tip}[\mathbf{X}(t)] \leq \delta_{max}, \qquad 0 \leq t \leq l$$

$$b(t) \geq 0, \qquad 0 \leq t \leq l$$

$$d(t) \geq 0, \qquad 0 \leq t \leq l$$



(b)

Here the design variables are functions of the length parameter t. This type of problem, where each design variable is a function of one or more parameters, is known as a *trajectory* or *dynamic optimization problem*.

Classification Based on the Physical Structure of the Problem

- Optimal Control Problem: is a mathematical programming problem involving a number of stages, where each stage evolves from the preceding stage in a prescribed manner. It is usually described by two types of variables: the control (design) and the state variables. The control variables define the system and govern the evolution of the system from one stage to the next, and the state variables describe the behavior or status of the system in any stage. The problem is to find a set of control or design variables such that the total objective function over all the stages is minimized subject to a set of constraints on the control and state variables.
- Non-Optimal Control Problem: The problems which are not optimal control problems are called non-optimal control problems.

Example 2

A rocket is designed to travel a distance of $12s$ in a vertically upward direction. The thrust of the rocket can be changed only at the discrete points located at distances of $0, s, 2s, 3s, \ldots, 12s$. If the maximum thrust that can be developed at point i either in the positive or negative direction is restricted to a value of F_i, formulate the problem of minimizing the total time of travel under the following assumptions:

1. The rocket travels against the gravitational force.
2. The mass of the rocket reduces in proportion to the distance traveled.
3. The air resistance is proportional to the velocity of the rocket.

Example 2 | Optimal Control Problem