Introduction to Engineering Optimization (ME6806)

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Module 1

Module 1

Introduction to Engineering Optimization

- Optimization is the act of obtaining the best result under given circumstances.
- Optimization can be defined as the process of finding the conditions that give the maximum or minimum of a function.
- The optimum seeking methods are also known as *mathematical programming techniques* and are generally studied as a part of operations research.
- *Operations research* is a branch of mathematics concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions.

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If a point $x*$ corresponds to the minimum value of the function $f(x)$, the same point also corresponds to the maximum value of the negative of the function, $-f(x)$. Thus optimization can be taken to mean minimization since the maximum of a function can be found by seeking the minimum of the negative of the same function.

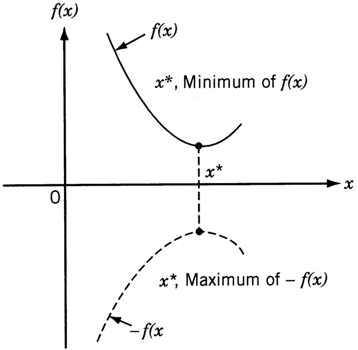

Figure: Minimum of $f(x)$ is same as maximum of $-f(x)$

The objective function will not change the optimum solution $x*$

1. Multiplication (or division) of $f(x)$ by a positive constant c.
2. Addition (or subtraction) of a positive constant c to (or from) $f(x)$.

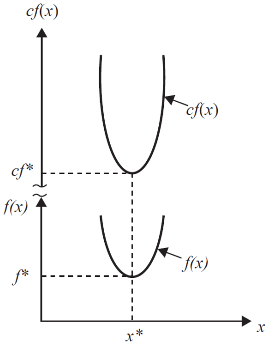

Figure: Optimum solution of $cf(x)$ or $c + f(x)$ same as that of $f(x)$

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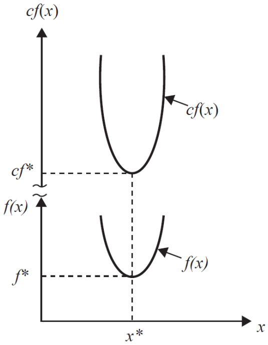

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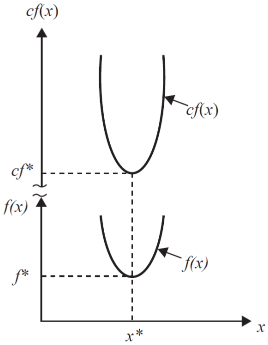

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Applications of Engineering Optimization I

Optimization, in its broadest sense, can be applied to solve any engineering problem:

1. Design of aircraft and aerospace structures for minimum weight
2. Finding the optimal trajectories of space vehicles
3. Design of civil engineering structures such as frames, foundations, bridges, towers, chimneys, and dams for minimum cost
4. Minimum-weight design of structures for earthquake, wind, and other types of random loading
5. Design of water resources systems for maximum benefit
6. Optimal plastic design of structures

Applications of Engineering Optimization II

7. Optimum design of linkages, cams, gears, machine tools, and other mechanical components
8. Selection of machining conditions in metal-cutting processes for minimum production cost
9. Design of material handling equipment, such as conveyors, trucks, and cranes, for minimum cost
10. Design of pumps, turbines, and heat transfer equipment for maximum efficiency
11. Optimum design of electrical machinery such as motors, generators, and transformers
12. Optimum design of electrical networks
13. Shortest route taken by a salesperson visiting various cities during one tour

Applications of Engineering Optimization III

14. Optimal production planning, controlling, and scheduling
15. Analysis of statistical data and building empirical models from experimental results to obtain the most accurate representation of the physical phenomenon
16. Optimum design of chemical processing equipment and plants
17. Design of optimum pipeline networks for process industries
18. Selection of a site for an industry
19. Planning of maintenance and replacement of equipment to reduce operating costs
20. Inventory control

Applications of Engineering Optimization IV

21. Allocation of resources or services among several activities to maximize the benefit
22. Controlling the waiting and idle times and queueing in production lines to reduce the costs
23. Planning the best strategy to obtain maximum profit in the presence of a competitor
24. Optimum design of control systems

End of Module 1

Module 2

Module 2

Statement of an Optimization Problem I

An optimization or a mathematical programming problem can be stated as follows:

Find $\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{Bmatrix}$ which minimizes $f(\mathbf{X})$ subject to the constraints

$$g_i(\mathbf{X}) \leq \mathbf{0} \qquad i = 0, 1, 2, \ldots \text{m}$$
$$l_j(\mathbf{X}) = \mathbf{0} \qquad j = 0, 1, 2, \ldots \text{p}$$

where **X** is an n-dimensional vector called the *design vector*, $f(\mathbf{X})$ is termed the *objective function*, and g_i and l_j are known as *enquality* and *equality* constraints respectively. The number of variables *n* and the number of constraints *m* and /or *p* need not be related in any way.

Statement of an Optimization Problem II

Find $\mathbf{X} = \begin{Bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{Bmatrix}$ which minimizes $f(\mathbf{X})$

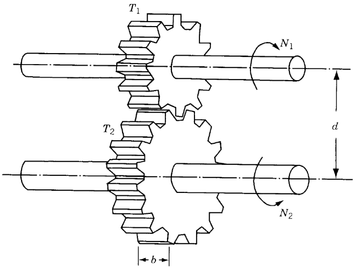
Such problems are called unconstrained optimization problems.

Design Vector

- Any engineering system or component is defined by a set of quantities some of which are viewed as variables during the design process.
- Certain quantities are usually fixed at the outset and these are called preassigned parameters.
- All the other quantities are treated as variables in the design process and are called design or decision variables $x_i, i = 1, 2, \ldots, n$. The design variables are collectively represented as a *design vector*

Design Vector I

- Consider the design of the gear pair characterized by its face width b, number of teeth T_1 and T_2, center distance d, pressure angle ψ, tooth profile, and material.
- If center distance d, pressure angle ψ, tooth profile, and material of the gears are fixed in advance, these quantities can be called *preassigned parameters*.
    - The remaining quantities can be collectively represented by a design vector **X** = $[x_1, x_2, x_3]^T = [b, T_1, T_2]^T$



Design Vector II

- If there are no restrictions on the choice of b, T_1, and T_2, any set of three numbers will constitute a design for the gear pair.
- If an n-dimensional Cartesian space with each coordinate axis representing a design variable x_i (i = 1, 2, . . . , n) is considered, the space is called the *design variable space* or simply *design space*.
- Each point in the n-dimensional design space is called a design point and represents either a possible or an impossible solution to the design problem.
  - the design point $[1.0, 20, 40]^T$, represents a possible solution, and
  - the design point $[1.0, -20, 40.5]^T$, represents an impossible solution.

Design Constraints