Operations Research

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LINEAR PROGRAMMING

- Linear Programming Problem
- Properties of LPs
- LP Solutions
- Graphical Solution
- Introduction to Sensitivity Analysis

Linear Programming (LP) Problem

- A mathematical programming problem is one that seeks to maximize or minimize an objective function subject to constraints.
- If both the objective function and the constraints are linear, the problem is referred to as a linear programming problem.
- Linear functions are functions in which each variable appears in a separate term raised to the first power and is multiplied by a constant (which could be 0).
- Linear constraints are linear functions that are restricted to be "less than or equal to", "equal to", or "greater than or equal to" a constant.

Linear Programming (LP) M

- There are five common types of decisions in which LP may play a role
  - Product mix
  - Production plan
  - Ingredient mix
  - Transportation
  - Assignment

Steps in Developing a Linear Programming (LP) Model

1) Formulation

2) Solution

3) Interpretation and Sensitivity Analysis

Steps in Formulating LP Problems

1. Define the objective. (min or max)
2. Define the decision variables. (positive, binary)
3. Write the mathematical function for the objective.
4. Write a 1- or 2-word description of each constraint.
5. Write the right-hand side (RHS) of each constraint.
6. Write $\leq$, =, or $\geq$ for each constraint.
7. Write the decision variables on LHS of each constraint.
8. Write the coefficient for each decision variable in each constraint.

Properties of LP Models

1) Seek to minimize or maximize
2) Include "constraints" or limitations
3) There must be alternatives available
4) All equations are linear

LP Problems in: Product Mix

- Objective

  To select the mix of products or services that results in maximum profits for the planning period

- Decision Variables

  How much to produce and market of each product or service for the planning period

- Constraints

  Maximum amount of each product or service demanded; Minimum amount of product or service policy will allow; Maximum amount of resources available

Example LP Model Formulation:
The Product Mix Problem

Decision: How much to make of $\geq$ 2 products?

Objective: Maximize profit

Constraints: Limited resources

Example: Pine Furniture Co.

Two products: Chairs and Tables

Decision: How many of each to make this month?

Objective: Maximize profit

Pine Furniture Data

	Tables (per table)	Chairs (per chair)	Hours Available
Profit Contribution	$7	$5	
Carpentry	3 hrs	4 hrs	2400
Painting	2 hrs	1 hr	1000

Other Limitations:

- Make no more than 450 chairs
- Make at least 100 tables

Constraints:

- Have 2400 hours of carpentry time available

$$3T + 4C \leq 2400 \qquad \text{(hours)}$$

- Have 1000 hours of painting time available

$$2T + 1C \leq 1000 \qquad \text{(hours)}$$

More Constraints:

- Make no more than 450 chairs

$$C \le 450 \quad \text{(num. chairs)}$$

- Make at least 100 tables

$$T \ge 100 \quad \text{(num. tables)}$$

Nonnegativity:

Cannot make a negative number of chairs or tables

$$T \ge 0$$

$$C \ge 0$$

Model Summary

Max $7T + 5C$ (profit)

Subject to the constraints:

$3T + 4C \leq 2400$ (carpentry hrs)

$2T + 1C \leq 1000$ (painting hrs)

$C \leq 450$ (max # chairs)

$T \geq 100$ (min # tables)

$T, C \geq 0$ (nonnegativity)

Graphical Solution

- Graphing an LP model helps provide insight into LP models and their solutions.

- While this can only be done in two dimensions, the same properties apply to all LP models and solutions.

Carpentry

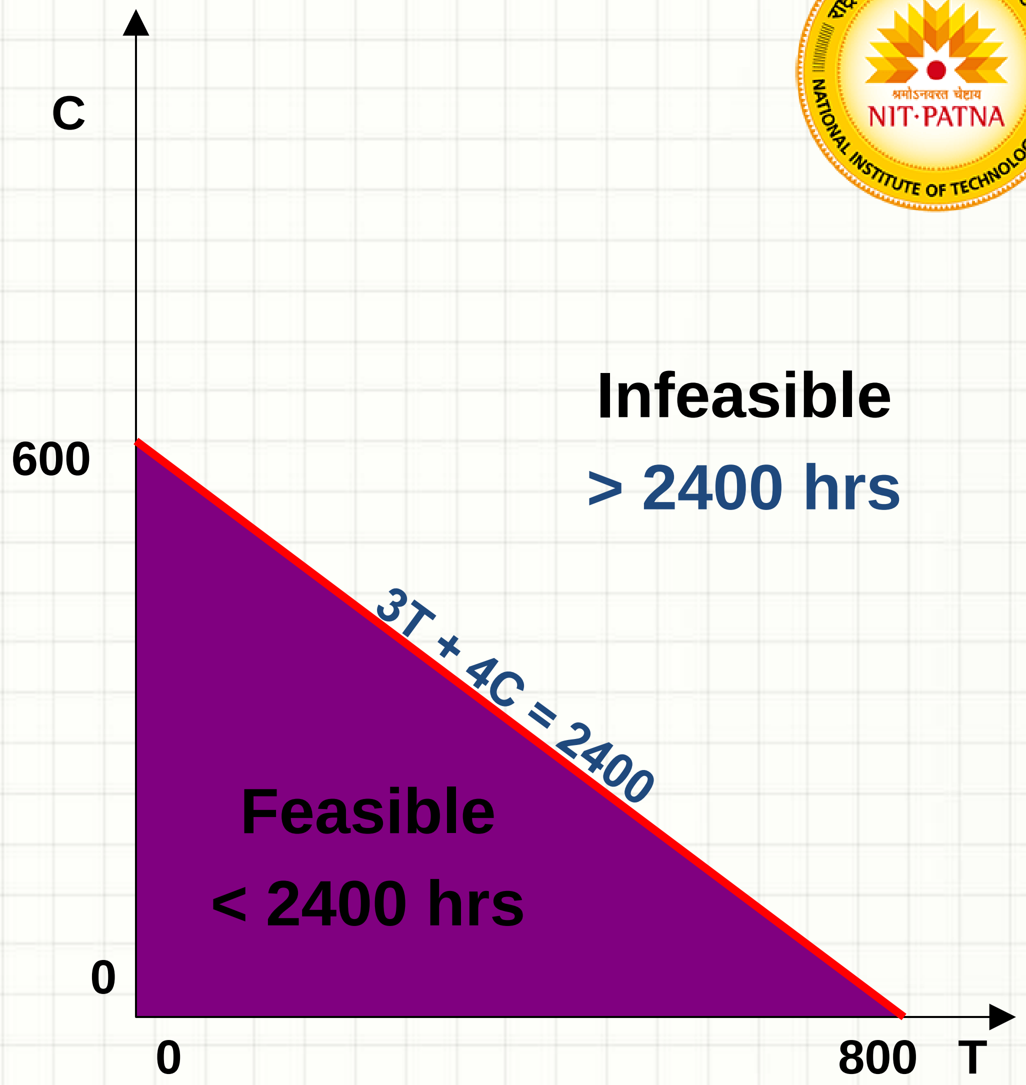
Constraint Line

$3T + 4C = 2400$

Intercepts

$(T = 0, C = 600)$

$(T = 800, C = 0)$



Painting

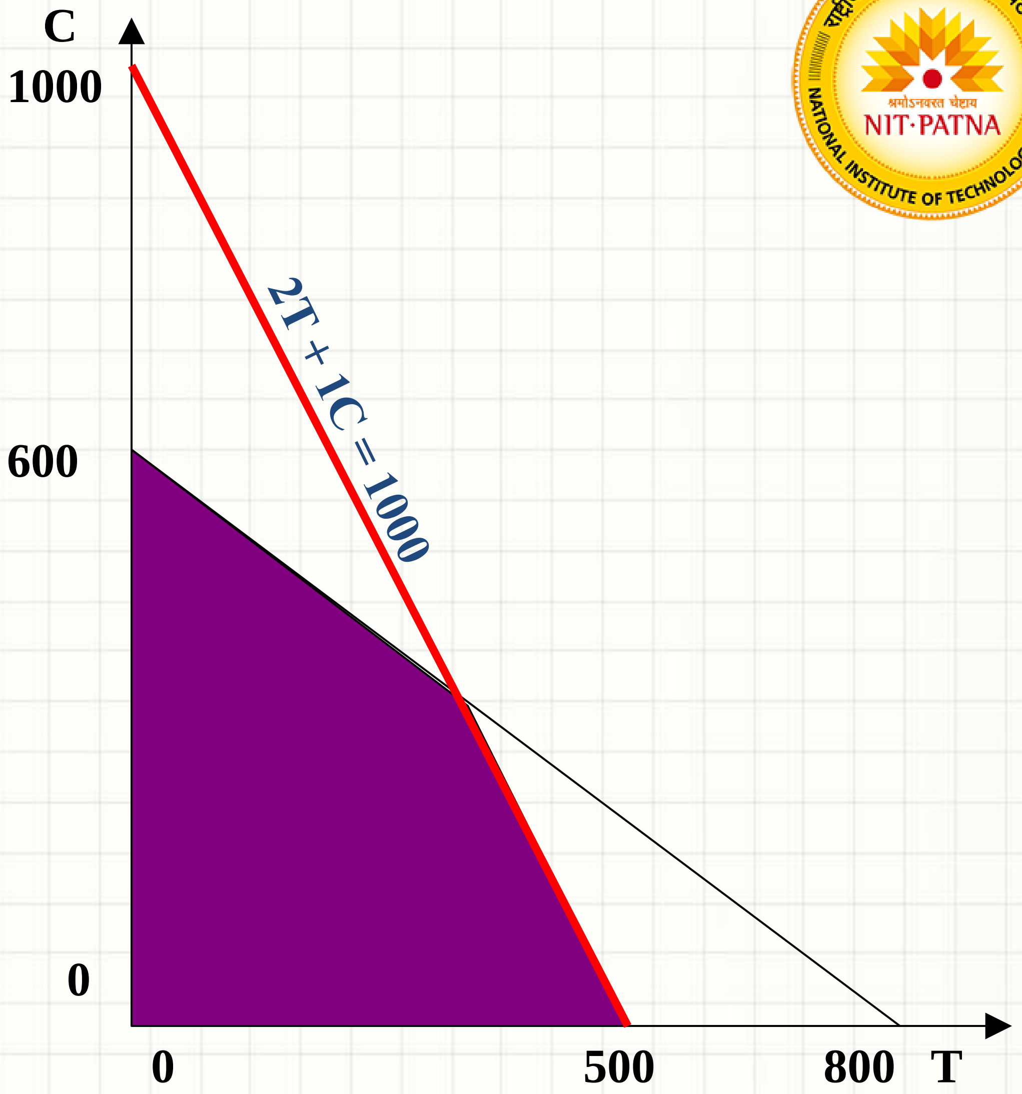
Constraint Line

2T + 1C = 1000

Intercepts

(T = 0, C = 1000)

(T = 500, C = 0)

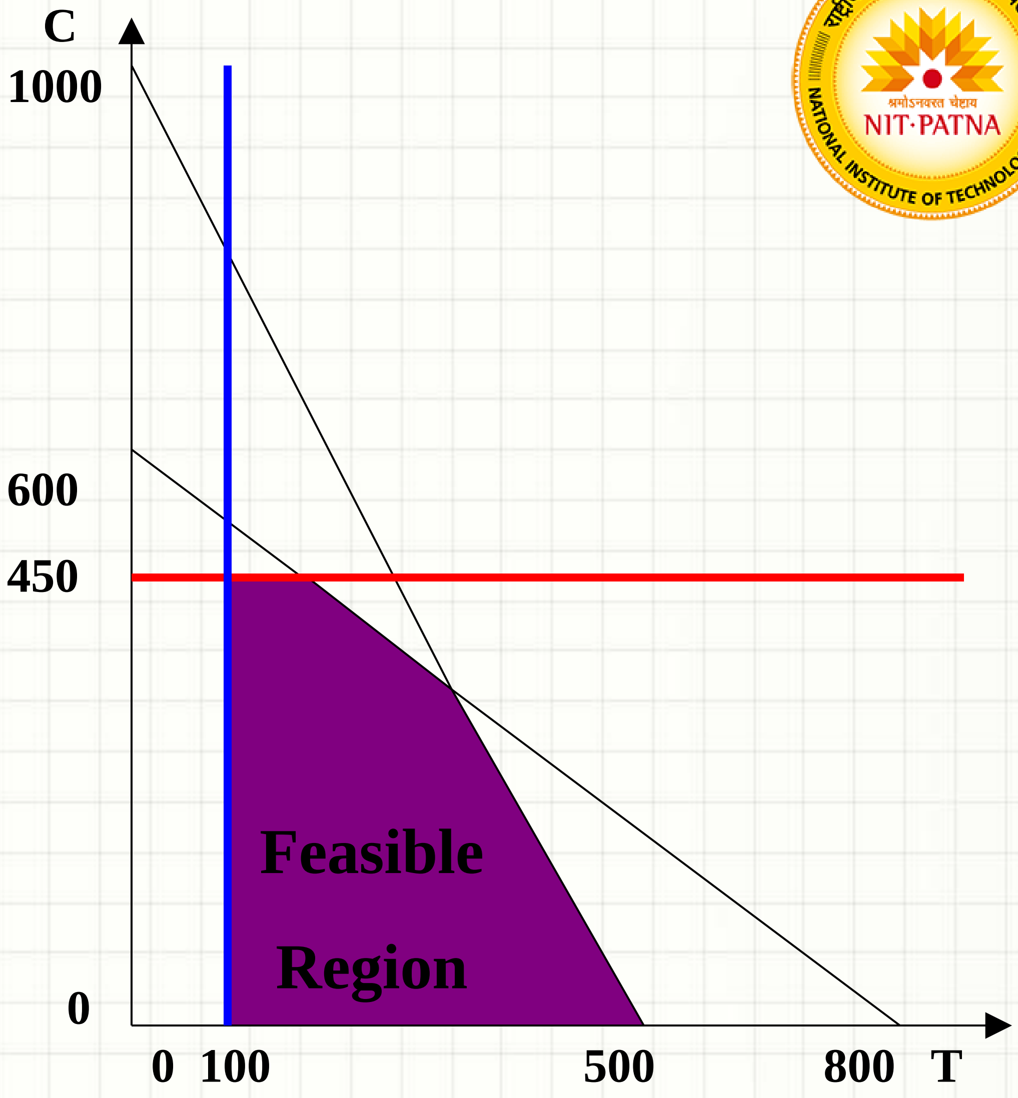


Max Chair Line

C = 450

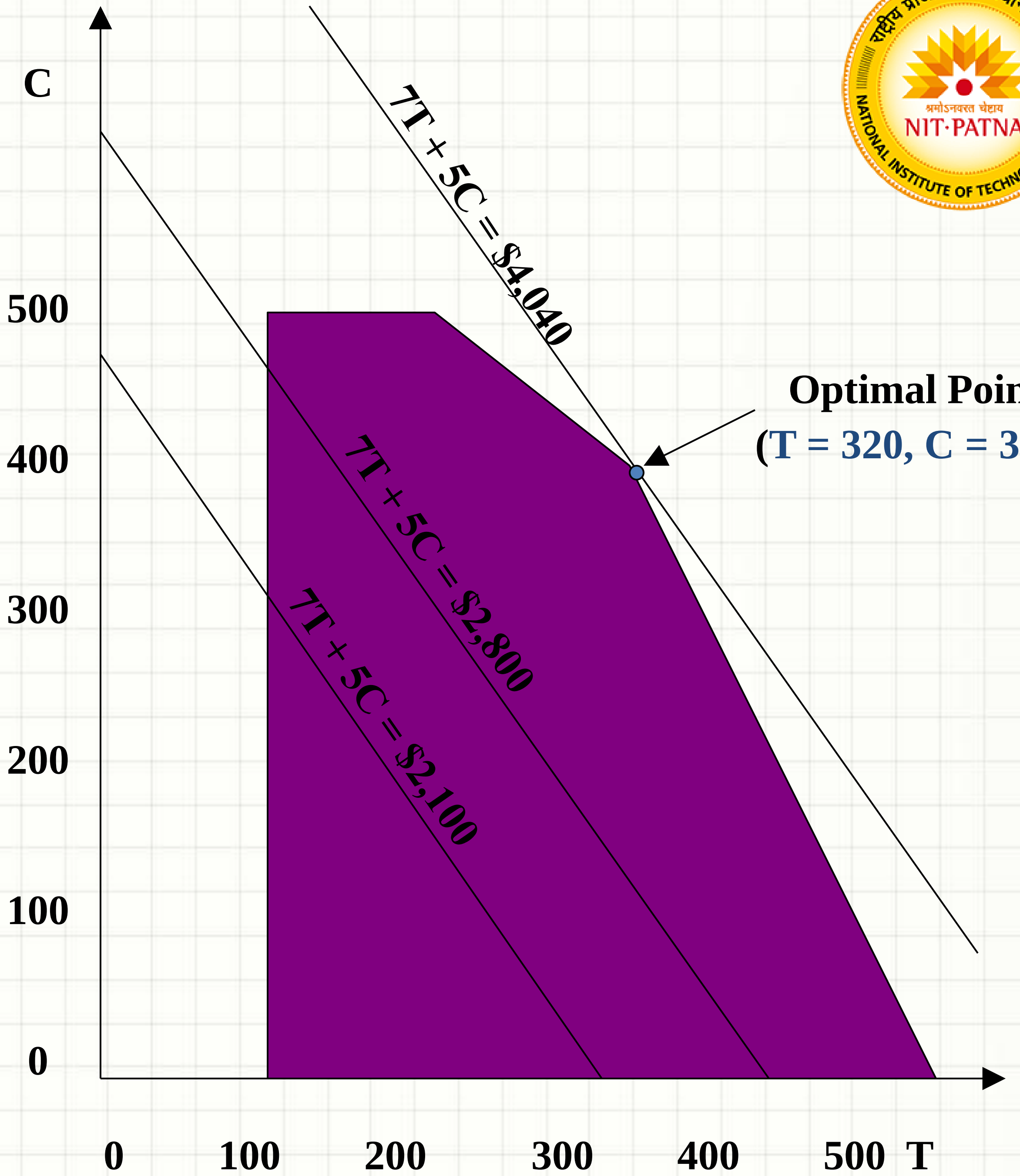
Min Table Line

T = 100



Objective Function Line

7T + 5C = Profit



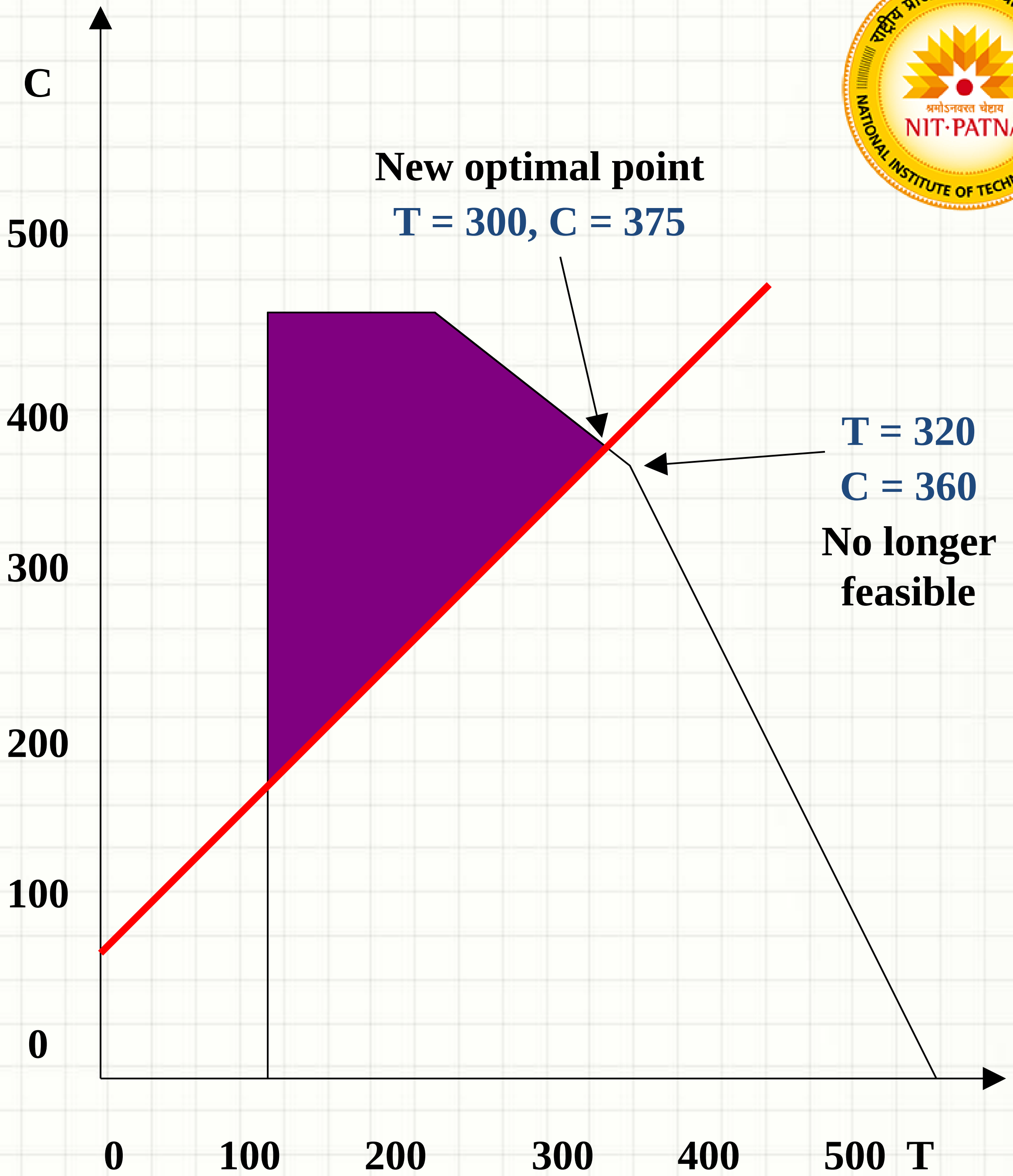
Additional Constraint

Need at least 75 more chairs than tables

$C > T + 75$

Or

$C - T > 75$



LP Characteristics

- **Feasible Region**: The set of points that satisfies all constraints
- **Corner Point Property**: An optimal solution must lie at one or more corner points
- **Optimal Solution**: The corner point with the best objective function value is optimal

Special Situation in LP

1. **Redundant Constraints** - do not affect the feasible region

   Example: $x \leq 10$

   $x \leq 12$

   The second constraint is redundant because it is *less* restrictive.

Special Situation in LP

2. **Infeasibility** – when no feasible solution exists (there is no feasible region)

Example: $x \leq 10$

$x \geq 15$

Special Situation in LP

3. **Alternate Optimal Solutions** – when there is more than one optimal solution

Max $2T + 2C$

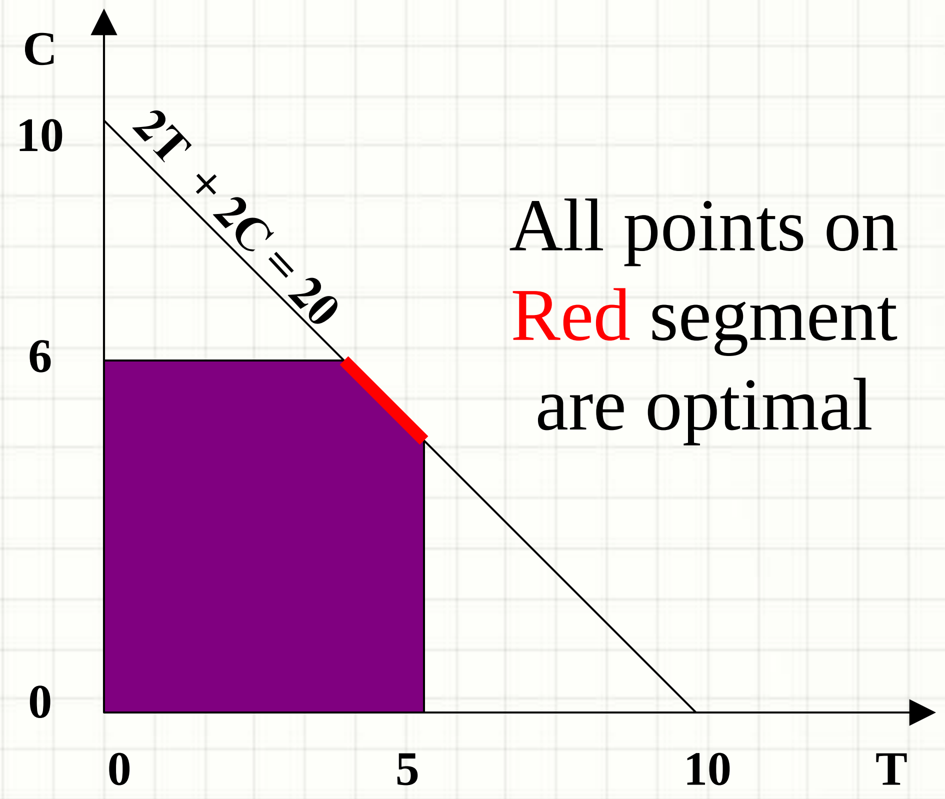
Subject to:

$T + C < 10$

$T < 5$

$C < 6$

$T, C > 0$



All points on Red segment are optimal

Special Situation in LP

4. **Unbounded Solutions** – when nothing prevents the solution from becoming infinitely large

Max 2T + 2C

Subject to:

2T + 3C > 6

T, C > 0